Module-08

Internal, Epicyclic and Other Special Gearing

Lecture-39

Gear Train, Internal and Epicyclic Gearing

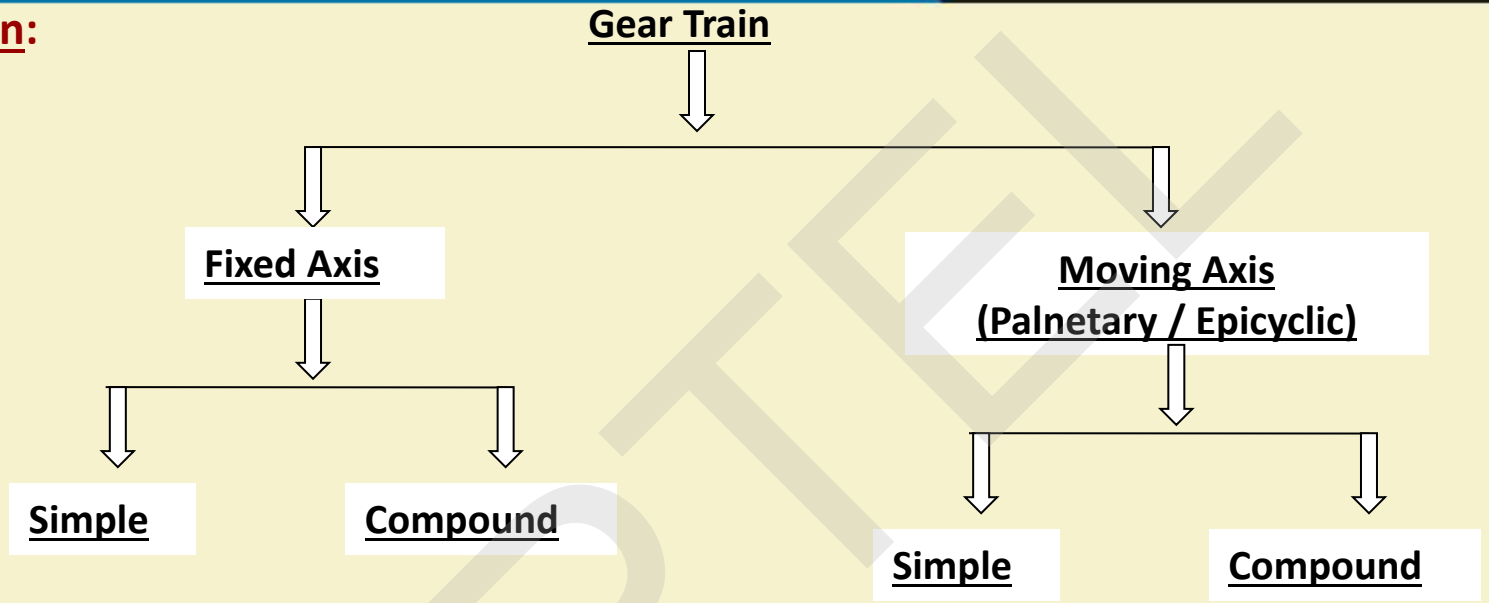
NPTEL

Prof. Rathindranath Maiti
Mechanical Engineering Department

Outline of the Lecture

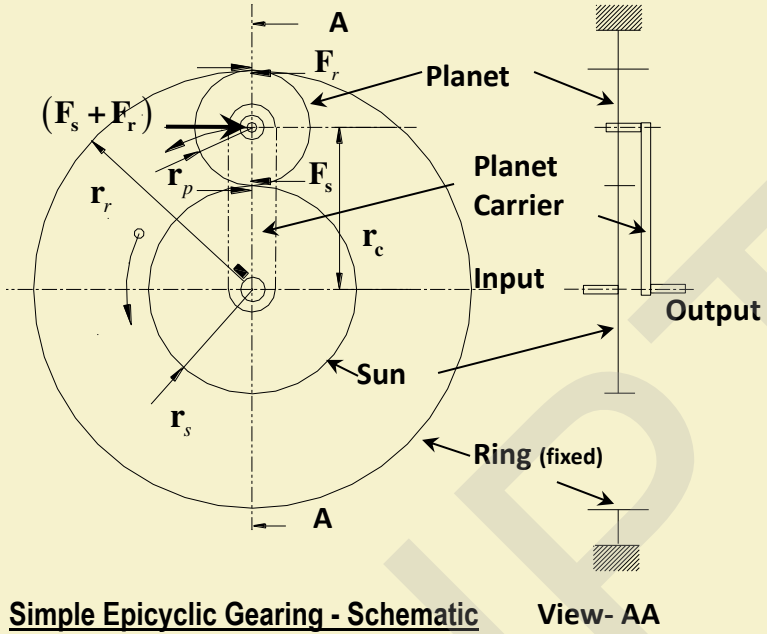
- Gear Train
  - Type of Epicyclic Gearing
    - Kinematic Relations
      - Simple and Compound Gear Train
        - Transmission Ratio
      - Two Gear Epicyclic Drive
        - Planocentri Gear and Cycloid Speed Reducer

Gear Train:



In case of moving axis gearing velocity is different from pitch line velocity, with respect to reference frame, which are equal in fixed axis gearing.

Epicyclic (or Planetary) Gearing (Simple Train):



Simple Epicyclic Gearing - Schematic View- AA

In general Input is Sun Gear, Planet Carrier is Output whereas Ring Gear remains fixed.

Input (Sun Gear) Torque:

$$T_i = T_s = F_s \times r_s$$

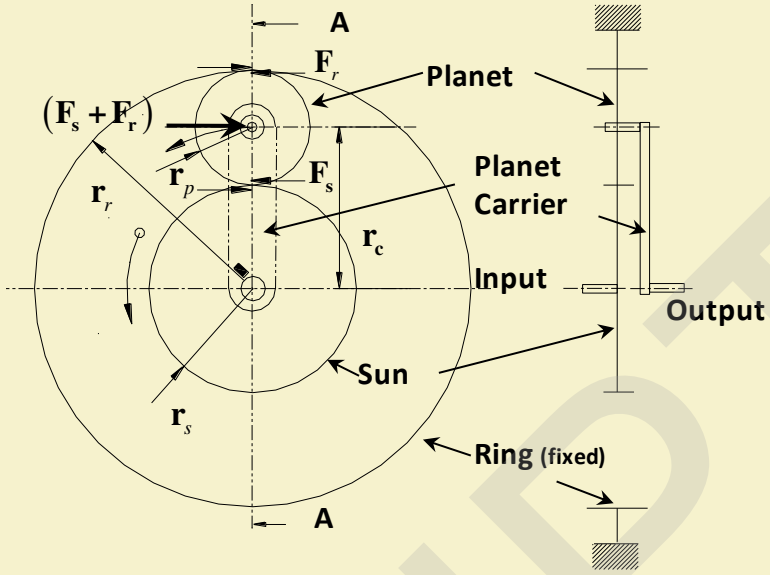
Output (Planet Carrier/Arm) Torque:

$$T_o = T_c = (F_s + F_r) \times r_c$$

But, $F_s = F_r$

and, $r_c = A = (r_s + r_r)/2$

Epicyclic (or Planetary) Gearing (Simple Train) (Contd…) :



Simple Epicyclic Gearing - Schematic View- AA

Therefore, output Torque

$$T_o = T_c = F_s(r_s + r_r)$$

Transmission ratio=Output Torque/Input Torque

or, $$i_t = \frac{T_o}{T_i} = \frac{F_s(r_s + r_r)}{F_s r_s}$$

i.e., $$i_t = 1 + \frac{r_r}{r_s}$$

In terms of teeth number,

$$i_t = 1 + \frac{Z_r}{Z_s}$$

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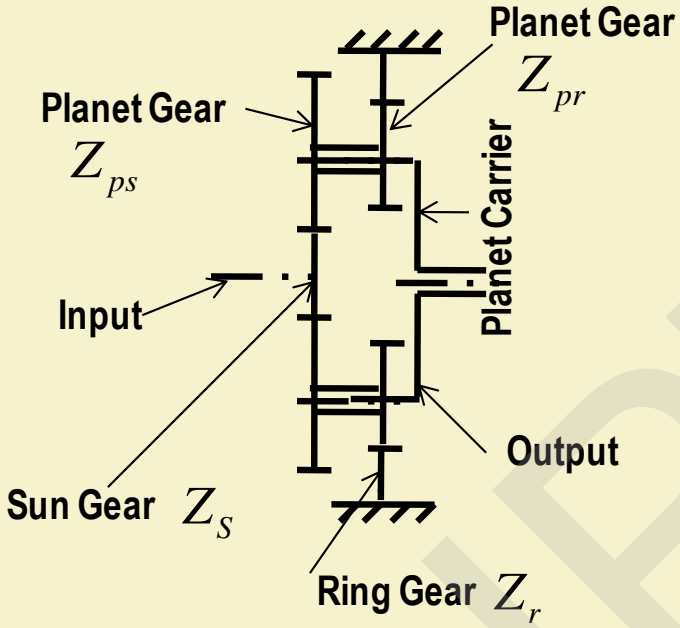
Epicyclic (or Planetary) Gearing (Compound) (Contd…) :



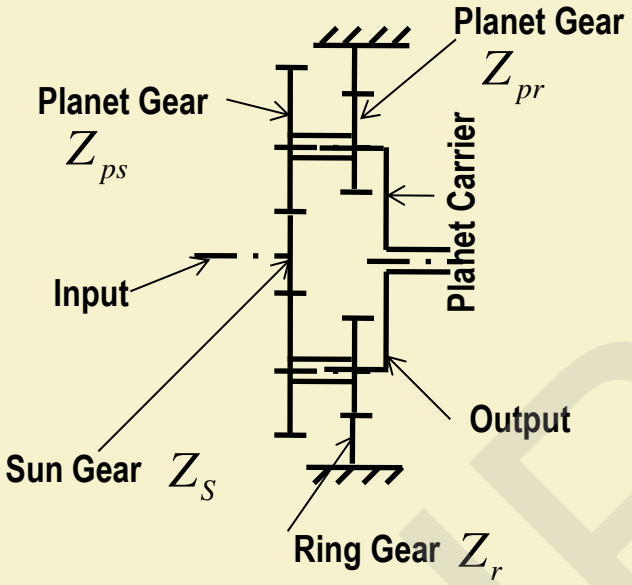
Compound Epicyclic Gearing (Side view)

In case of compound epicyclic gear,

$$i_t = 1 + \frac{Z_r}{Z_s} \times \frac{Z_{ps}}{Z_{pr}}$$

It is to be noted that in first case the gearing is simple internal gear train and number of teeth in planet pinion does not come into transmission ratio directly.

Epicyclic (or Planetary) Gearing (Compound) (Contd…) :



Compound Epicyclic Gearing (Side view)

In terms of compound gearing-

The transmission ratio in Simple Epicyclic gearing is:

$$i_t = 1 + \frac{Z_p}{Z_s} \times \frac{Z_r}{Z_p} = 1 + \frac{Z_r}{Z_s}$$

$$Z_r = Z_s + 2Z_p$$

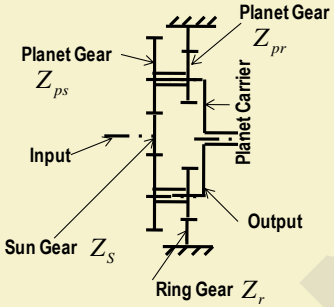
Epicyclic (or Planetary) Gearing (Compound) (Contd…) :

Derivation of transmission ratio by Tabular method :

Operation	Speed				
	Planet Meshing Ring	Planet Meshing Sun	Sun	Arm Planet Carrier	Ring
Arm fixed. Ring given -1 Rotation.	$-\frac{Z_r}{Z_{pr}}$	$-\frac{Z_r}{Z_{pr}}$	$\frac{Z_r}{Z_{pr}} \times \frac{Z_{ps}}{Z_s}$	0	-1
The whole unit given +1 Rotation	+1	+1	+1	+1	+1
Total Resultant	$1-\frac{Z_r}{Z_{pr}}$	$1-\frac{Z_r}{Z_{pr}}$	$1+\frac{Z_r}{Z_{pr}} \times \frac{Z_{ps}}{Z_s}$	+1	0

Epicyclic (or Planetary) Gearing (Compound) (Contd…) :

Operation	Speed				
	Planet Meshing Ring	Planet Meshing Sun	Sun	Arm Planet Carrier	Ring
Arm fixed. Ring given -1 Rotation.	$-\frac{Z_r}{Z_{pr}}$	$-\frac{Z_r}{Z_{pr}}$	$\frac{Z_r}{Z_{pr}}\times\frac{Z_{ps}}{Z_s}$	0	-1
The whole unit given +1 Rotation	+1	+1	+1	+1	+1
Total Resultant	$1-\frac{Z_r}{Z_{pr}}$	$1-\frac{Z_r}{Z_{pr}}$	$1+\frac{Z_r}{Z_{pr}}\times\frac{Z_{ps}}{Z_s}$	+1	0



Compound Epicyclic Gearing (Side view)

Derivation of transmission ratio by Tabular method :

Therefore, Transmission ratio=Input Rotation (Speed)/Output Rotation (Speed)

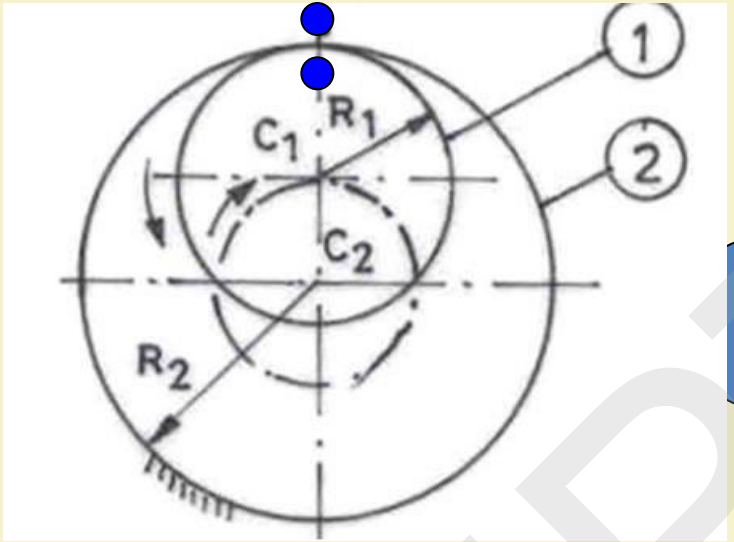
or, $$i_t = \frac{1+\frac{Z_r}{Z_{pr}}\times\frac{Z_{ps}}{Z_s}}{1}$$

i.e., $$i_t = 1+\frac{Z_r}{Z_{pr}}\times\frac{Z_{ps}}{Z_s}$$

In case simple train, $i = 1+\frac{Z_r}{Z_s}$ as, $Z_{ps} = Z_{pr} = Z_p$

(Two Gear Epicyclic Drive – Ring Gear Fixed, Cam input and Star Gear Output)

Working Principle & Gearing Ratio

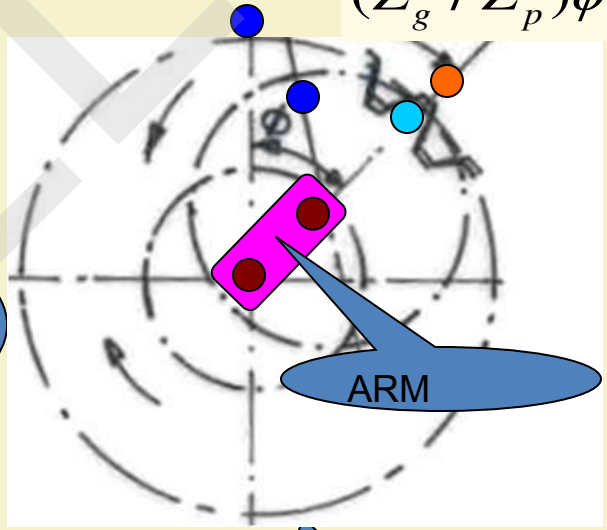


Rotation of Whole Unit (About Central Axes)

$$\left[\frac{\phi}{\phi-(Z_g/Z_p)\phi}\right]$$

Net Rotation of Inner one (About its own Axes)

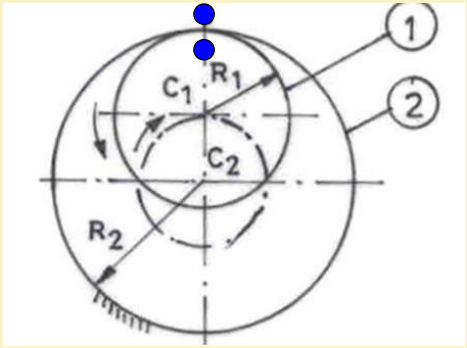
$(Z_g/Z_p)\phi$

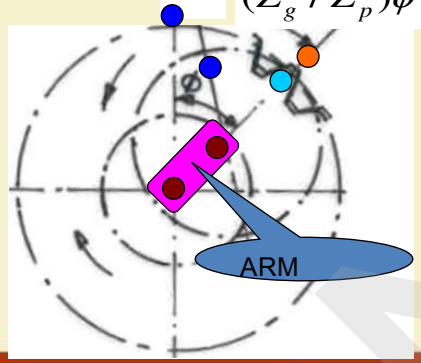


Net Rotation of Outer one = 0

(Two Gear Epicyclic Drive – Ring Gear Fixed, Cam input and Star Gear Output)

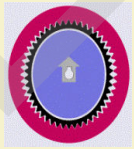
Working Principle & Gearing Ratio

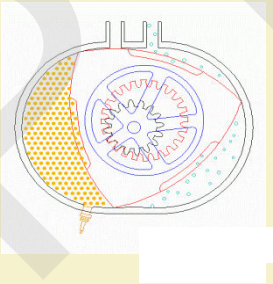




Transmission Ratio

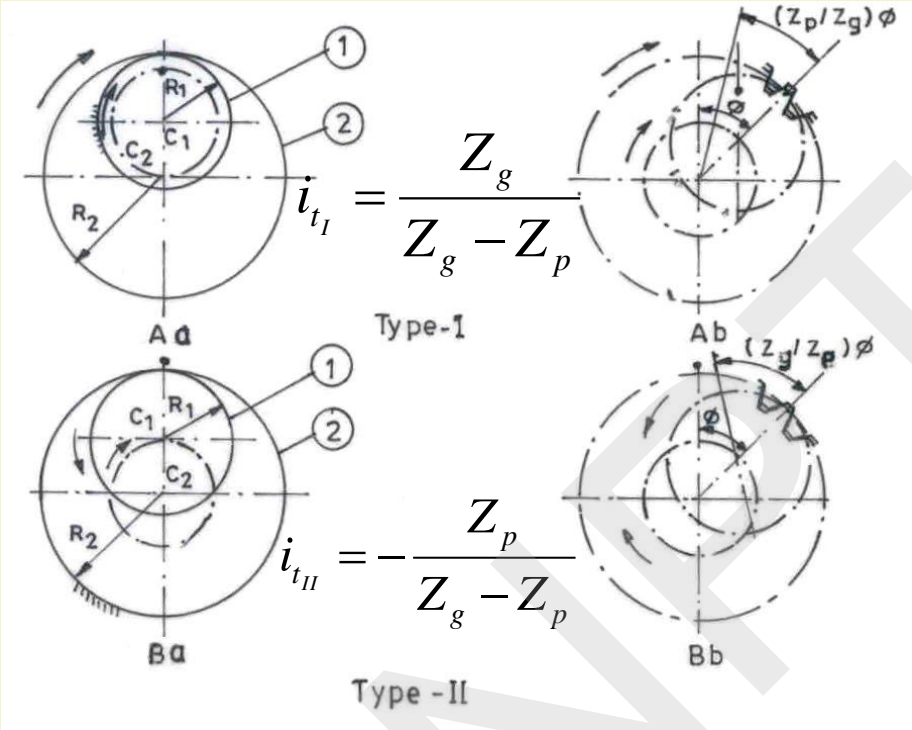
$$i_{t_{II}} = -\frac{Z_p}{Z_g - Z_p}$$

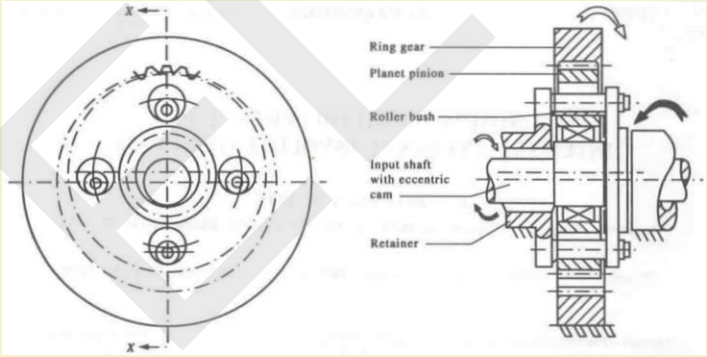
Both &  Fall in this category



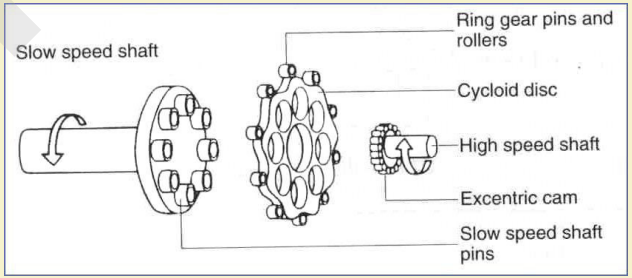
$$\left[\frac{\phi}{\phi - (Z_p / Z_g)\phi}\right] \quad i_{t_I} = \frac{Z_g}{Z_g - Z_p}$$

Two Gear Epicyclic Drives (Summary & Application):



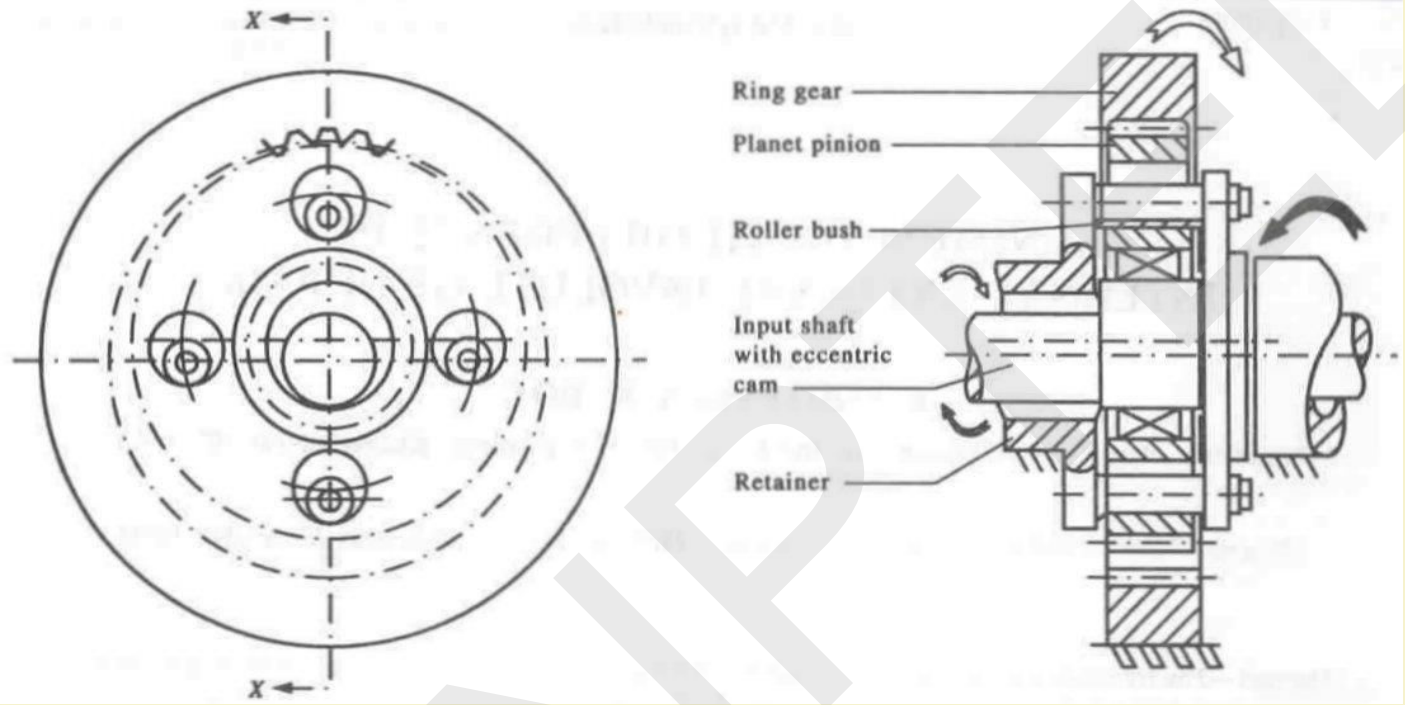


Two Gear Epicyclic Drive (Planocentri Gear)



Two Gear Epicyclic Drive (Cycloid Speed Reducer)

Two Gear Epicyclic Drives (Summary & Application):



Two Gear Epicyclic Drive (Planocentri Gear)

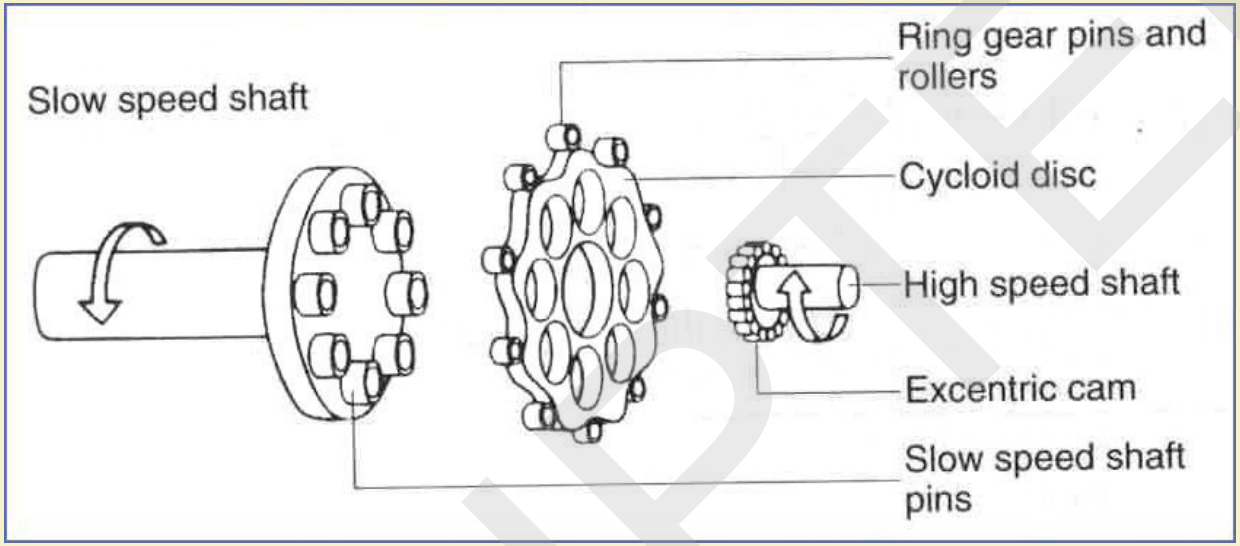
Rotation of Pinion about its own axis constrained, Cam Input, Ring Output

$$i_{t_I} = \frac{Z_g}{Z_g - Z_p}$$

Ring Fixed, Cam Input, Pinion Output

$$i_{t_{II}} = -\frac{Z_p}{Z_g - Z_p}$$

Two Gear Epicyclic Drives (Summary & Application):



Two Gear Epicyclic Drive (Cycloid Speed Reducer)

Rotation of Pinion about its own axis constrained, Cam Input, Ring Output

$$i_{t_I} = \frac{Z_g}{Z_g - Z_p}$$

Ring Fixed, Cam Input, Pinion Output

$$i_{t_{II}} = -\frac{Z_p}{Z_g - Z_p}$$

Thank You

Module-08

Internal, Epicyclic and Other Special Gearing

Lecture-40

Harmonic Drive Gear

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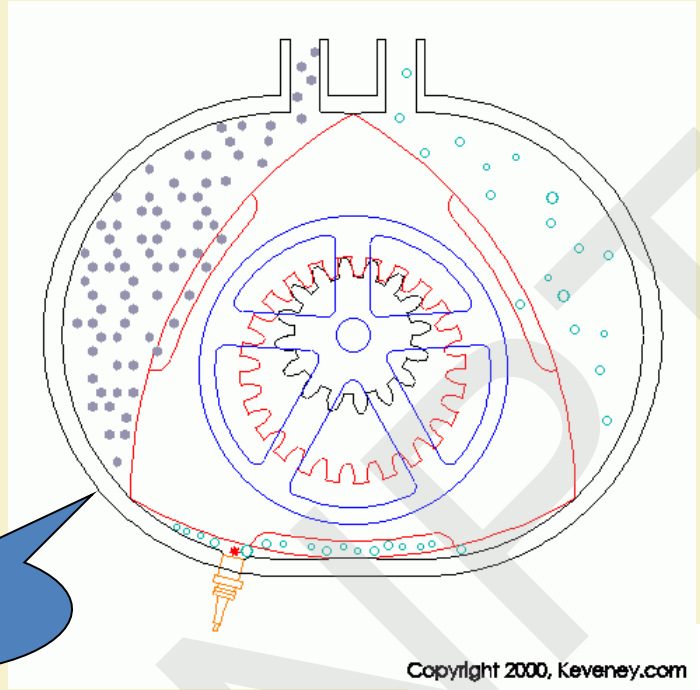
Mechanical Engineering Department

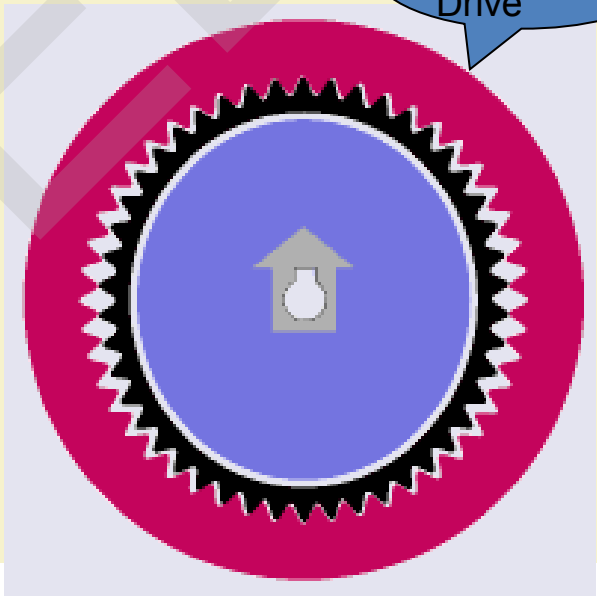
Outline of the Lecture

- Harmonic Drive, Wankel Engine & Orbit Motor in motion
  - Working Principle and Kinematic Relations
    - Components of Harmonic Drive
      - Gearing Transmission Ratio
  - Transformation of Two-Gear Epicyclic Drive to Harmonic Drive
    - Problem with Conventional Harmonic Drives & Remedies
      - Application Example of Harmonic Drives
  - Advantages- Harmonic Drives
    - Introduction to Tooth Tip Interference

Two Different units following same kinematic principle

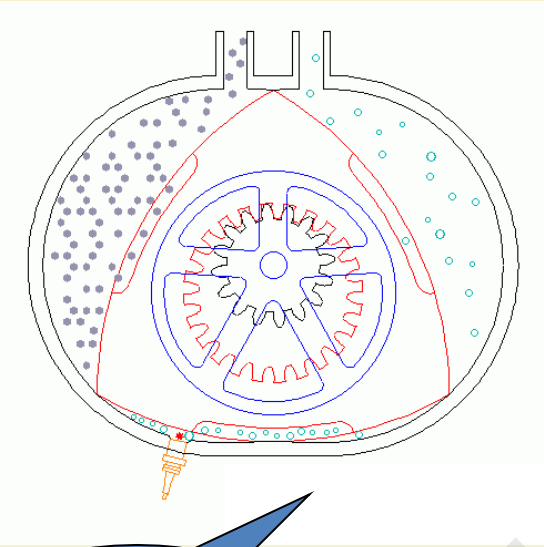


Wankel Engine



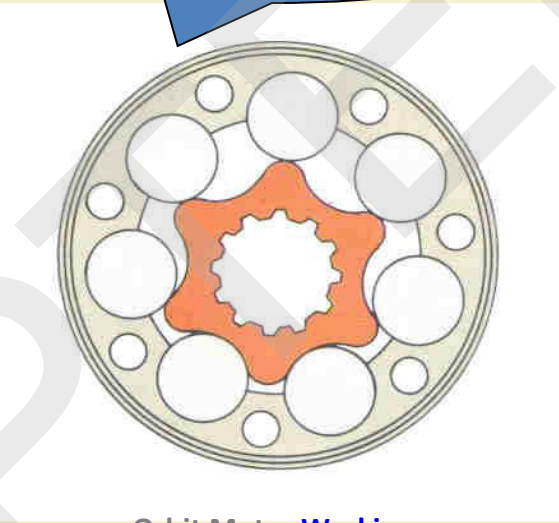
Harmonic Drive

Two Different units following same kinematic principle

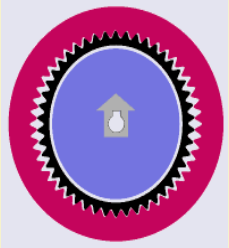


Wankel Engine

Orbit Motor
Gerotor Pump/Motor

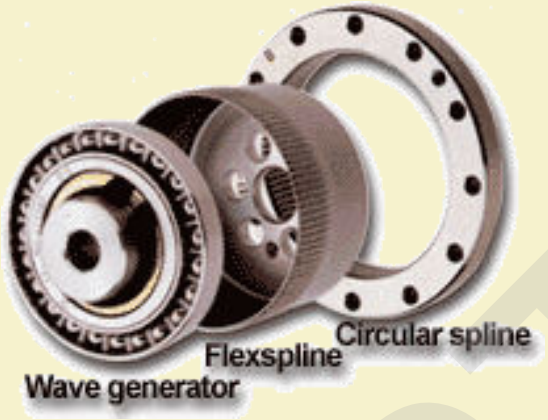


Orbit Motor Working



Harmonic Drive

What is a Harmonic Drive ?



a. Two gear epicyclic drive with a very high speed reduction.

b. Components:

   i. Flex spline

   ii. Circular spline (Ring gear)

   iii. Wave generator (Cam & Bearing assembly)

c. The tooth difference between the flex spline & the circular spline is two.

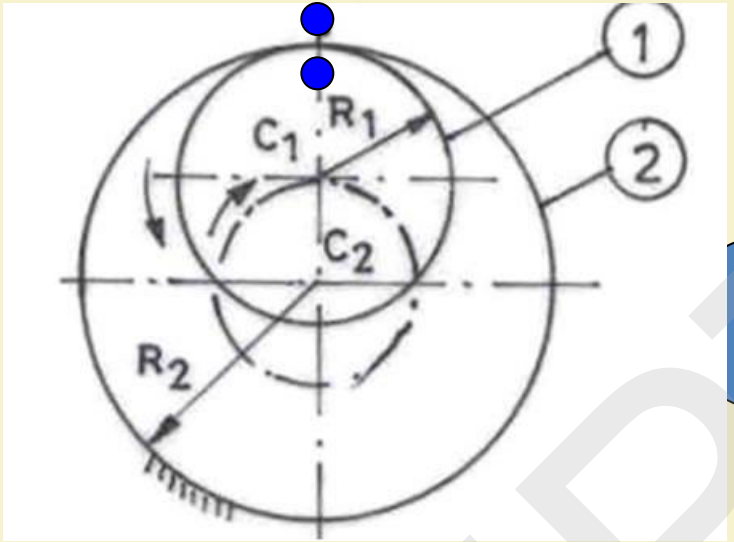
d. Transmission ratio

$$= -Z_p/(Z_g-Z_p)$$

a. [Ring fixed, Cam input, Flexspline Output]

(Two Gear Epicyclic Drive – Ring Gear Fixed, Cam input and Star Gear Output)

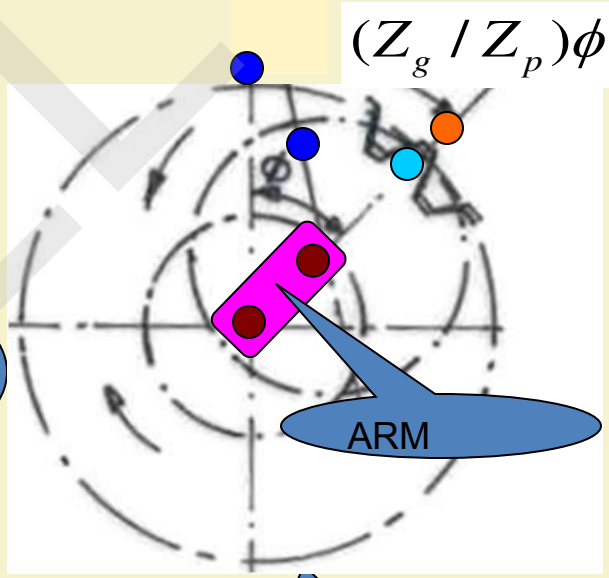
Working Principle & Gearing Ratio (Recapitulation)



Rotation of Whole Unit (About Central Axes)

$$\left[\frac{\phi}{\phi-(Z_g/Z_p)\phi}\right]$$

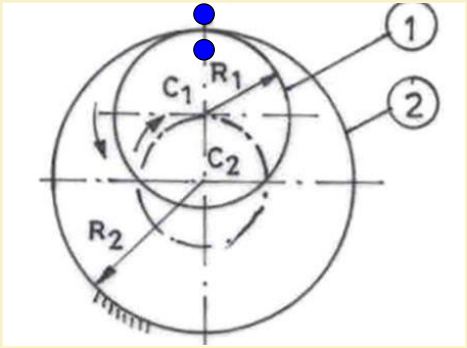
Net Rotation of Inner one (About its own Axes)



Net Rotation of Outer one = 0

(Two Gear Epicyclic Drive – Ring Gear Fixed, Cam input and Star Gear Output)

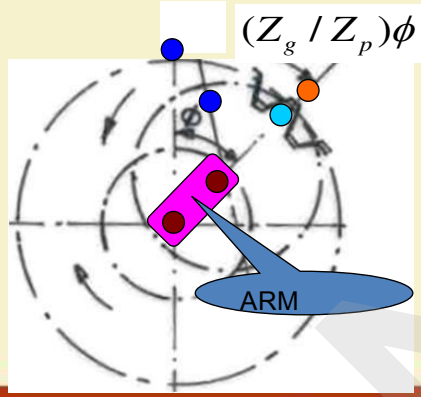
Working Principle & Gearing Ratio (Recapitulation)

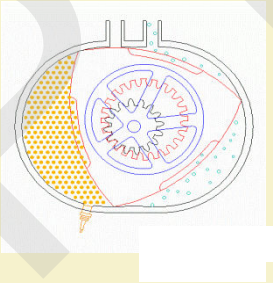


Transmission Ratio

$$i_{t_{II}} = -\frac{Z_p}{Z_g - Z_p}$$

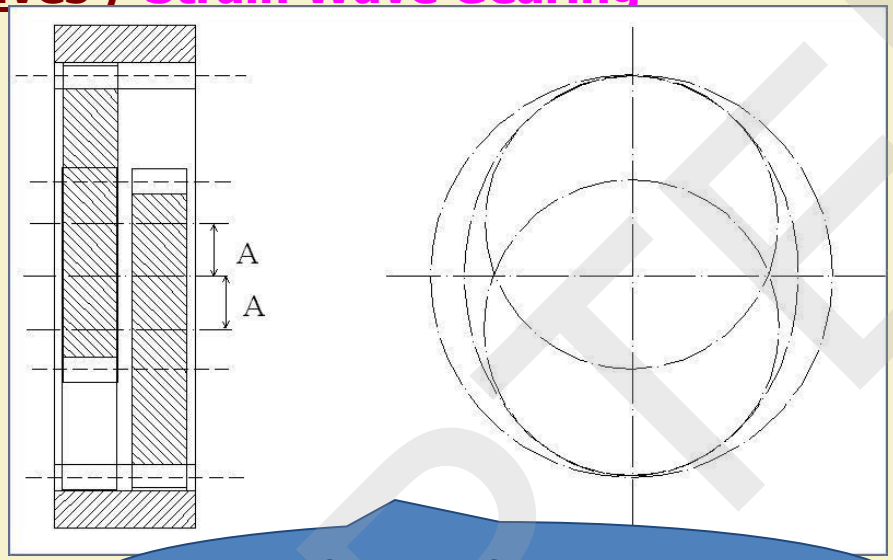
Both & Fall in this category

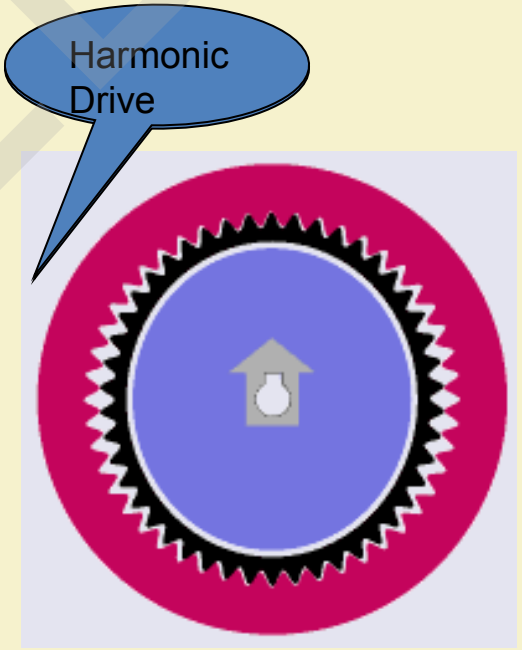




$$\left[\frac{\phi}{\phi - (Z_p / Z_g)\phi}\right] \quad i_{t_I} = \frac{Z_g}{Z_g - Z_p}$$

Harmonic Drives / Strain Wave Gearing

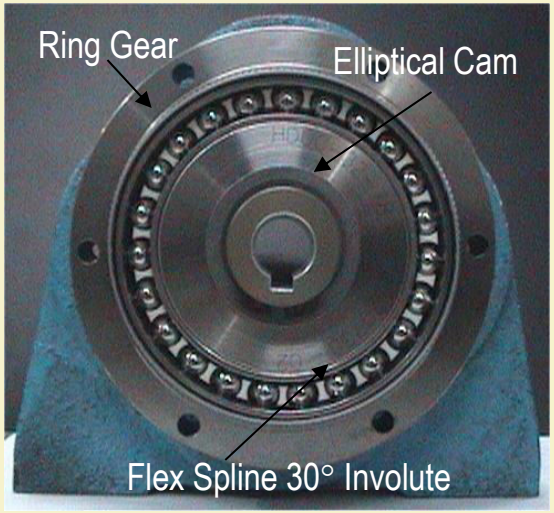




Maiti, R. (2004): A Novel Harmonic Drive with Pure Involute Tooth Gear Pair.

ASME Journal of Mechanical Design. 126(1):178-182.

Problem with conventional design



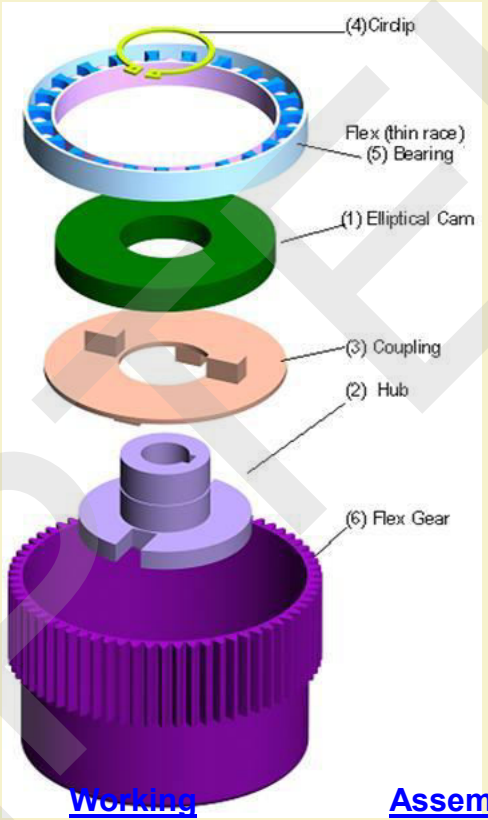
I. The cam is elliptical – Conjugacy at teeth contacts are affected
II. Involute - Standard and most popular profile can not be used
III. To maintain conjugacy tooth profile is synthesized – Gear pair become match part- Performance is sensitive to centre distance and accuracy

HARMONIC DRIVE: TR=78
(FS Output, CS fixed)
Z_p=156 & Z_g=158
Module= 0.529 mm (48 DP)

Conventional: CR=0.78

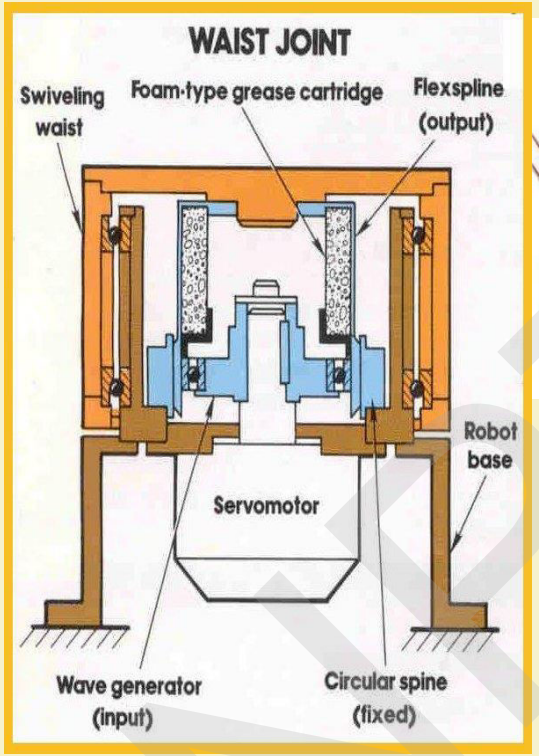
Exploded view of HDs

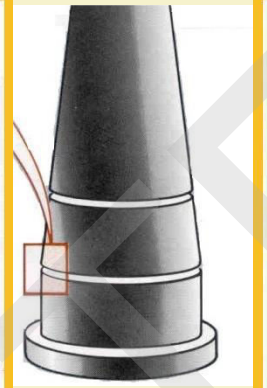
Conventional Commercial Unit

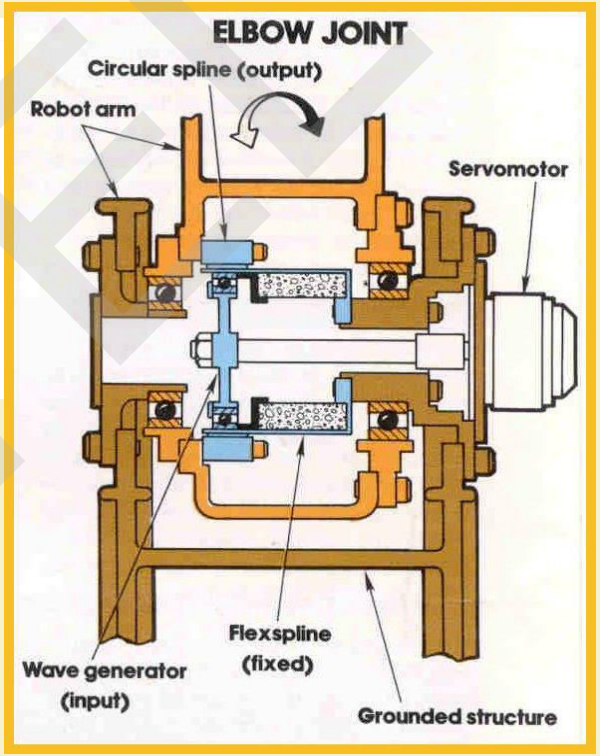


Working Assembly

Application Examples of Harmonic Drive

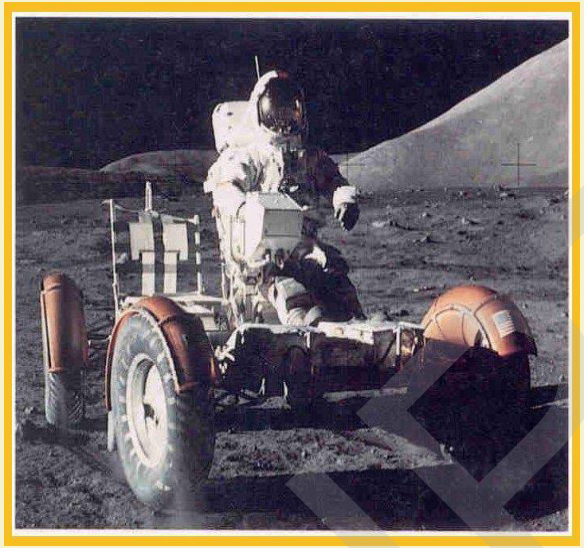






Application examples of Harmonic Drives (Contd.....)

Continuous Duty/No Downtime- When it has to stand up.

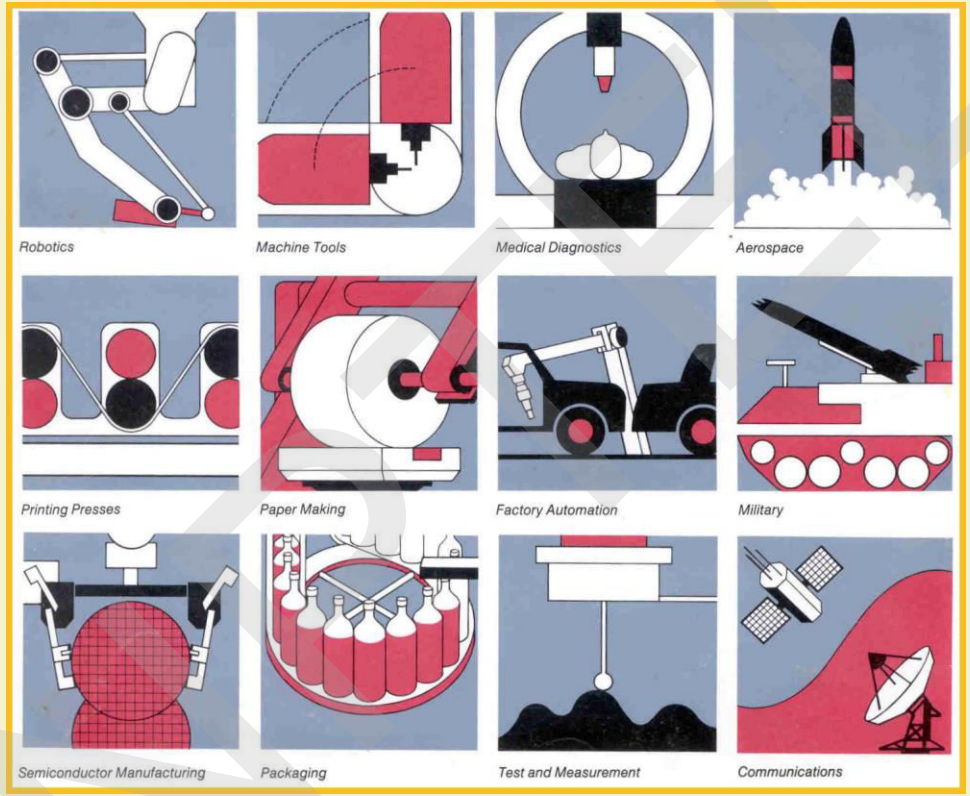


High Reliability- When it has to work.

Precision and Repeatability- When it has to drive precisely.

Compactness- When it has to fit into a small space.

Application examples of Harmonic Drives (Contd.....)

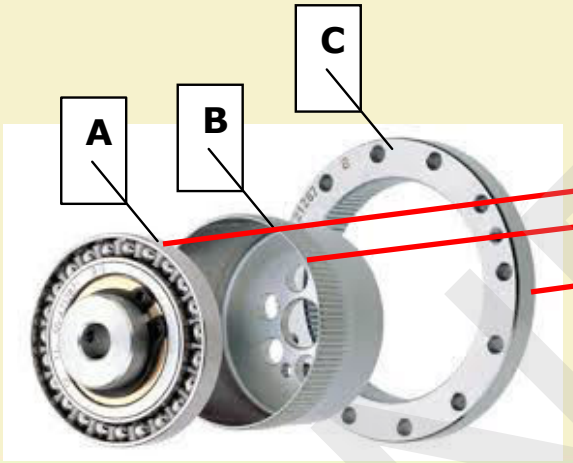


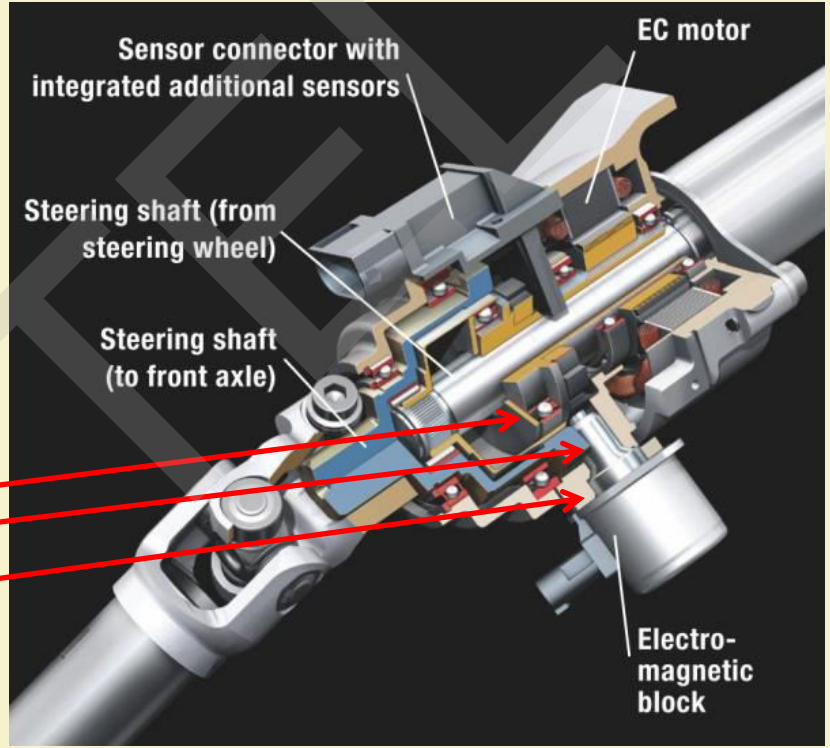
Application examples of Harmonic Drives (Contd.....)

Harmonic Drive used in Brain Surgery for Precision Work

Application of Harmonic Drive in Power Steering

Motor is concentric (in-line)



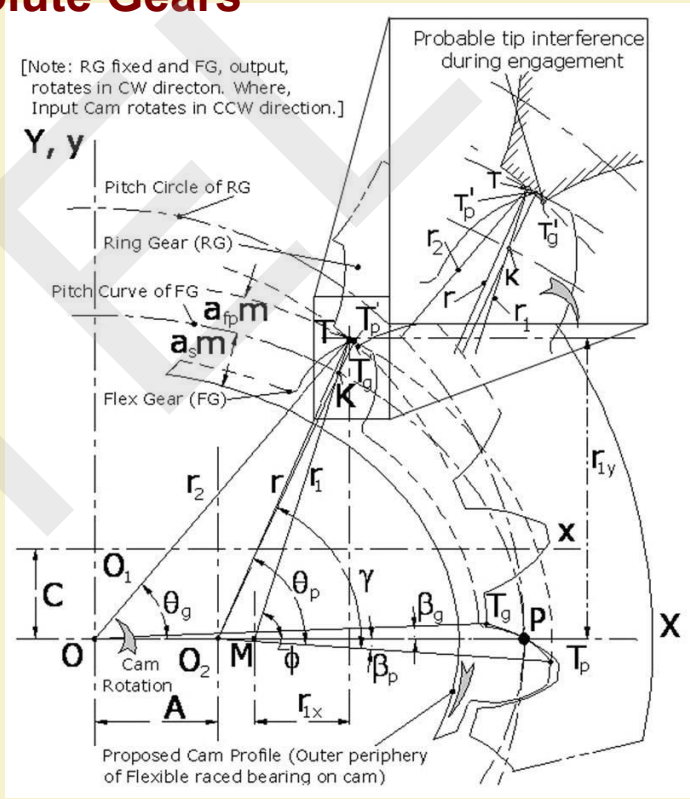


Active front steering

Advantages of Harmonic Drives

- Outstanding positional accuracy and repeatability.
- In line, high single stage reduction.
- Low or zero backlash configuration.
- Exceptional torque to weight ratio.
- High efficiency.
- Configuration versatility.
- Long life and high reliability.
- High precision.
- Smooth running.
- High torsional stiffness.
- Back Driving (also can be used for increase in speed.)

Major Problem in Internal Gearing with Involute Gears -Tooth Tip Interference



How the tooth tip interference is avoided- to be discussed later.

References

To know more about Harmonic Drives following published work and referred articles in these references to be followed.

Journal

Maiti, R. & Roy, A. K. (1996): Minimum possible tooth difference in involute internal-external gear pair. Mechanism and Machine Theory, 31 (1):475-485.

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Sahoo, V. & Maiti, R. (2016) : Evidence of Secondary Tooth Contact in Harmonic Drive, with Involute Toothed Gear Pair, through Experimental and Finite Element Analyses of Stresses in Flex-Gear Cup, Proceedings of the IMechE (UK), Journal of Mechanical Engineering Science, Part C. *(online* DOI: 10.1177/0954406216682541*).*

Sahoo, V. & Maiti, R. (2016) : Static load sharing by tooth pairs in contact in internal involute spur gearing with thin rimmed pinion. Proceedings of IMechE (UK), Journal of Mechanical Engineering Science, Part C. 2*30(4): 485–499,* [Also reprinted in the *Special Issue: Power Transmission with Gears*, 230 (7-8)] DOI: 10.1177/0954406215618424.

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Conferences

Routh, B. & Maiti, R. (2011): On a Gearing Problem in Conventional Harmonic Drives with Involute Toothed Gear Set. Procd. of the ASME IDETC/CIE 2011 conference, August 29-31, 2011, Washington, DC, USA.

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Routh, B., **Maiti, R.** & Ray, A. K. (2014): Aspects of lubrication at the wave generator -flexspline interface in strain wave gearing units, Procd. of International Gear Conference 2014, 26-28 August : Lyon, France.

Sahoo, V. and **Maiti, R**. (2016): State of stress in strain wave gear Flexspline cup on insertion of drive Cam -experiment and analysis, Proceedings of the World Congress on Engineering 2016 (London, UK) Vol II , WCE 2016. pp. 966–971.

Thank You

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Mechanical Engineering Department

Module-08

Internal, Epicyclic and Other Special Gearing

Lecture-41

Reverted Gearing and Differential Gearing

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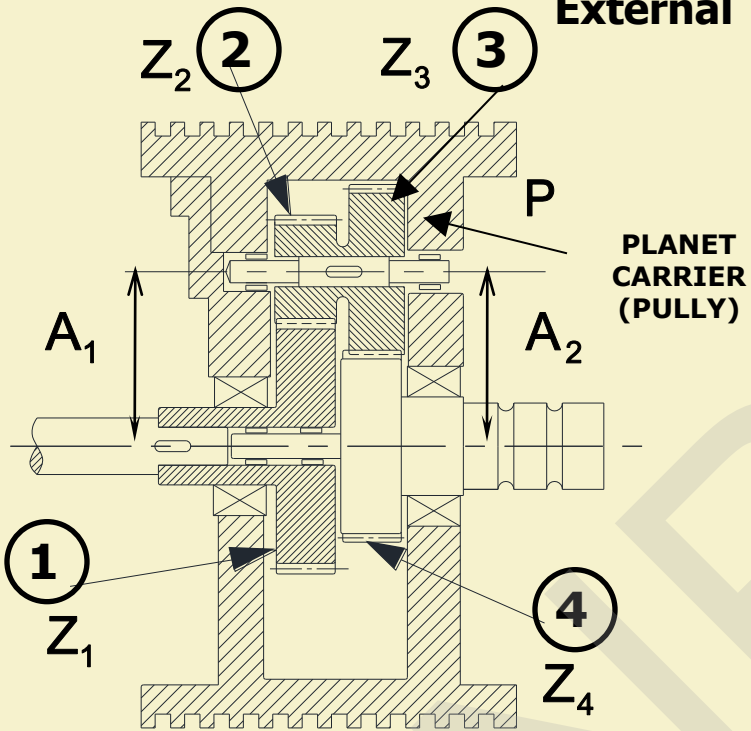
Outline of the Lecture

- Reverted Gearing
- Possible – Input-Output
- Transmission Ratio
- Workout Example
- Differential Gearing with Reverted Gearing
- Conventional Differential Gearing
- Comparison- Reverted Gearing & Differential Gearing

Reverted Gearing :

Reverted Gearing is an Epicyclic or Planetary Gear Drive with External Toothed Gears.



In general the Planet Carrier is Input and there may be three possible outputs:

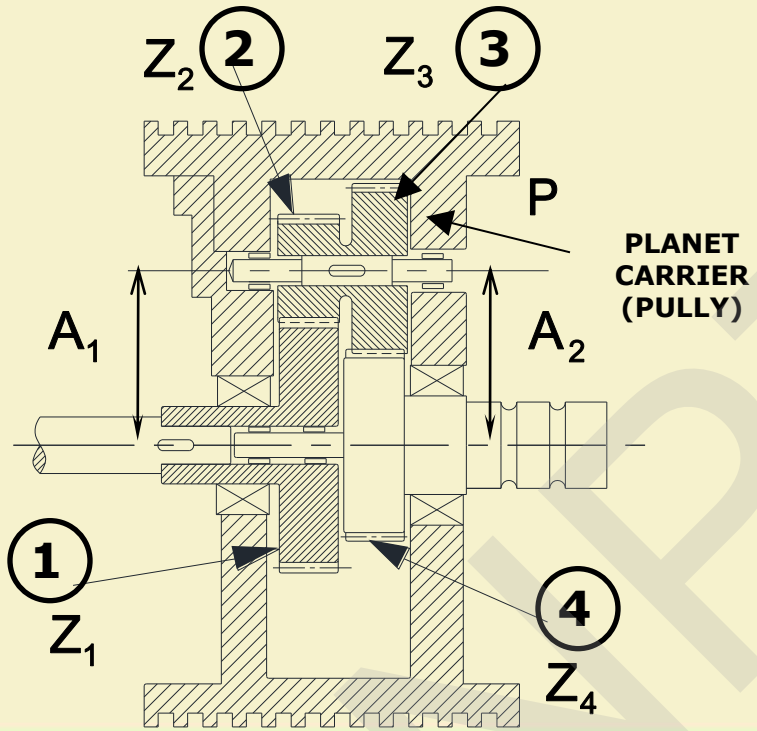
(i) Gear 1 is fixed & Gear 4 is output,

(ii) Gear 4 is fixed & Gear 1 is output,

(iii) Both Gear & Gear 4 are output.

Clearly such units can be used as differential gear unit.

Reverted Gearing (contd...) :



Basic Conditions:
Centre Distance-

$$A_1 = A_2$$

Or,

$$(Z_1 + Z_2)m_1 / 2$$
$$= (Z_3 + Z_4)m_2 / 2$$

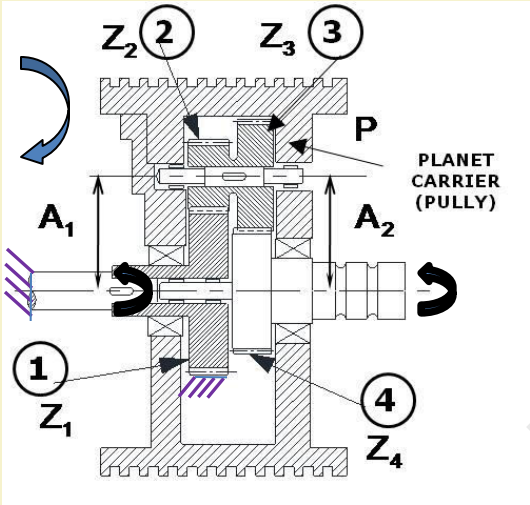
Usually (modules) $m_1 = m_2$

Therefore,

$$(Z_1 + Z_2) = (Z_3 + Z_4)$$

Reverted Gearing (contd...) :

Tabular method to find Transmission Ratio:



Planet Carrier is input

(i) Gear 1 is fixed & Gear 4 is output,

Operation	Speed (Direction of rotation – looking from left to right)				
	Planet Z_2	Planet Z_3	Gear 4 Z_4	Arm/ Planet Carrier	Gear 1 Z_1
Arm fixed. Gear 1 given -1 Rotation.	$\frac{Z_1}{Z_2}$	$\frac{Z_1}{Z_2}$	$-\frac{Z_1}{Z_2}\times\frac{Z_3}{Z_4}$	0	-1
The whole unit given +1 Rotation	+1	+1	+1	+1	+1
Total / Resultant	$1+\frac{Z_1}{Z_2}$	$1+\frac{Z_1}{Z_2}$	$1-\frac{Z_1}{Z_2}\times\frac{Z_3}{Z_4}$	+1	0

Reverted Gearing (contd...):

Tabular method to find Transmission Ratio:

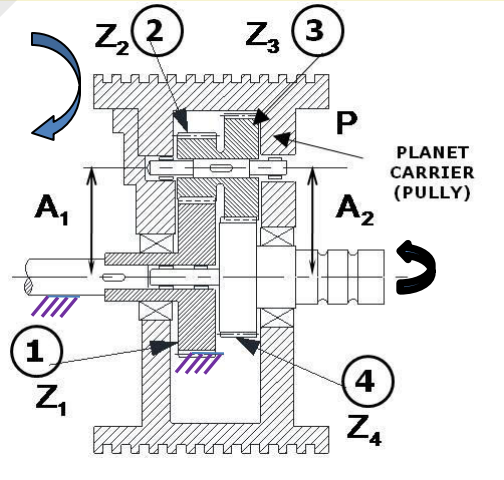
Operation	Speed (Direction of rotation – looking from left to right)				
	Planet Z_2	Planet Z_3	Gear 4 Z_4	Arm/ Planet Carrier	Gear 1 Z_1
Arm fixed. Gear 1 given -1 Rotation.	$\frac{Z_1}{Z_2}$	$\frac{Z_1}{Z_2}$	$-\frac{Z_1}{Z_2}\times\frac{Z_3}{Z_4}$	0	-1
The whole unit given +1 Rotation	+1	+1	+1	+1	+1
Total / Resultant	$1+\frac{Z_1}{Z_2}$	$1+\frac{Z_1}{Z_2}$	$1-\frac{Z_1}{Z_2}\times\frac{Z_3}{Z_4}$	+1	0

(i) Planet input, Gear 1 is fixed & Gear 4 is output:

$$i_1 = \frac{1}{1-\frac{Z_1}{Z_2}\times\frac{Z_3}{Z_4}} \quad \text{Or,} \quad i_1 = \frac{Z_2 Z_4}{Z_2 Z_4 - Z_1 Z_3}$$

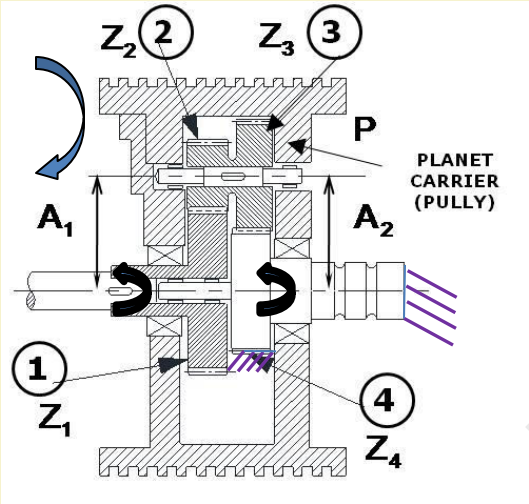
Transmission Ratio:

$$i_1 = \frac{\text{Input (Planet Carrier/Pulley)}}{\text{Output (Gear 4 Shaft)}}$$



Reverted Gearing (contd...) :

Tabular method to find Transmission Ratio:



Planet Carrier is input

(ii) Gear 4 is fixed & Gear 1 is output,

Operation	Speed (Direction of rotation – looking from left to right)				
	Planet Z_3	Planet Z_2	Gear 1 Z_1	Arm/ Planet Carrier	Gear 4 Z_4
Arm fixed. Gear 4 given -1 Rotation.	$\frac{Z_4}{Z_3}$	$\frac{Z_4}{Z_3}$	$-\frac{Z_2}{Z_1}\times\frac{Z_4}{Z_3}$	0	-1
The whole unit given +1 Rotation	+1	+1	+1	+1	+1
Total / Resultant	$1+\frac{Z_4}{Z_3}$	$1+\frac{Z_4}{Z_3}$	$1-\frac{Z_2}{Z_1}\times\frac{Z_4}{Z_3}$	+1	0

Reverted Gearing (contd...) :

Tabular method to find Transmission Ratio:

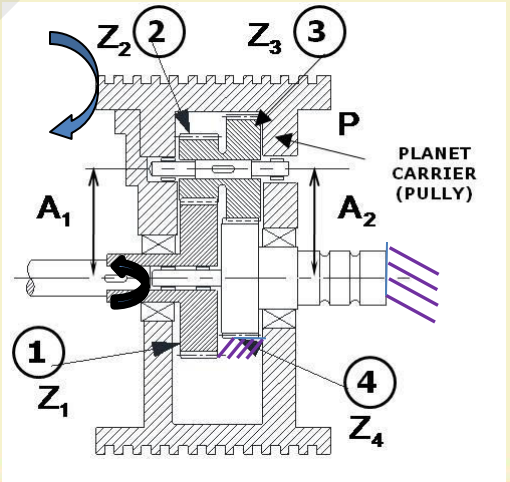
Operation	Speed (Direction of rotation – looking from left to right)				
	Planet Z_3	Planet Z_2	Gear 1 Z_1	Arm/ Planet Carrier	Gear 4 Z_4
Arm fixed. Gear 4 given -1 Rotation.	$\frac{Z_4}{Z_3}$	$\frac{Z_4}{Z_3}$	$-\frac{Z_2}{Z_1}\times\frac{Z_4}{Z_3}$	0	-1
The whole unit given +1 Rotation	+1	+1	+1	+1	+1
Total / Resultant	$1+\frac{Z_4}{Z_3}$	$1+\frac{Z_4}{Z_3}$	$1-\frac{Z_2}{Z_1}\times\frac{Z_4}{Z_3}$	+1	0

(i) Planet input, Gear 4 is fixed & Gear 1 is output:

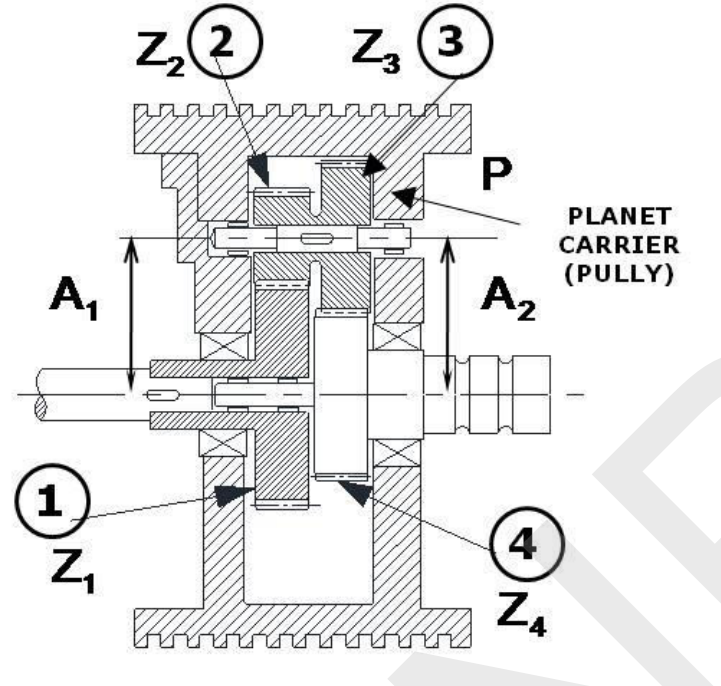
$$i_2 = \frac{1}{1-\frac{Z_2}{Z_1}\times\frac{Z_4}{Z_3}} \quad \text{Or,} \quad i_2 = \frac{Z_1 Z_3}{Z_1 Z_3 - Z_2 Z_4}$$

Transmission Ratio:

$$i_2 = \frac{\text{Input (Planet Carrier/Pulley)}}{\text{Output (Gear 1 Shaft)}}$$



Reverted Gearing (contd...):



Transmission Ratio (Summary):

(i) Planet input, Gear 1 is fixed & Gear 4 is output:

$$i_1 = \frac{Z_2 Z_4}{Z_2 Z_4 - Z_1 Z_3}$$

(ii) Planet input, Gear 4 is fixed & Gear 1 is output:

$$i_2 = \frac{Z_1 Z_3}{Z_1 Z_3 - Z_2 Z_4}$$

(iii) The unit can also be used as differential unit, with pulley / planet carrier as input and both Gear 1 and Gear 4 as output.

Gears 1 and 4 can also be internal gear.

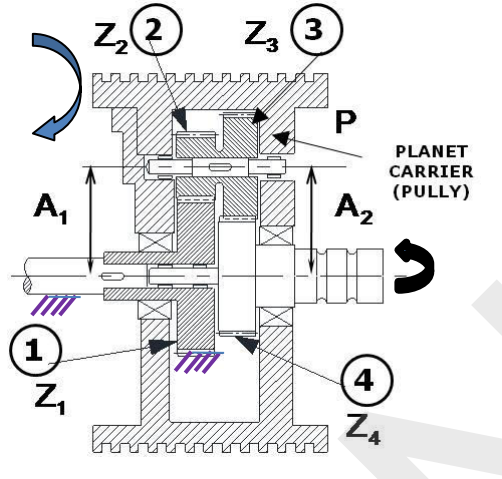
Reverted Gearing (contd...) :

Workout example:

Let $Z_1 = 101,\quad Z_2 = 98,\quad Z_3 = 99$ **and** $Z_4 = 100,$ **All with same module.**

Find two possible transmission ratios.

Solution: **The condition** $(Z_1 + Z_2) = (Z_3 + Z_4) = 199$ **is satisfied.**



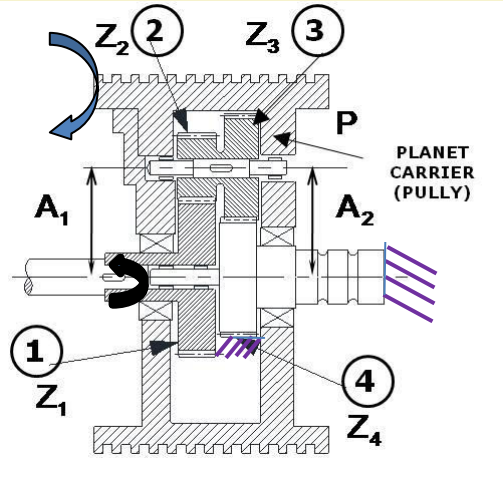
(i) For Planet input, Gear 1 is fixed & Gear 4 is output,

Transmission Ratio is :

$$i_1 = \frac{Z_2 Z_4}{Z_2 Z_4 - Z_1 Z_3} = -49.246$$

Reverted Gearing (contd…) :

Workout example (contd…) :



(ii) For Planet input, Gear 4 is fixed & Gear 1 is output, Transmission Ratio is:

$$i_2 = \frac{Z_1 Z_3}{Z_1 Z_3 - Z_2 Z_4} = +50.201$$

'+' sign indicates same directions of rotations and '-' sign indicates opposite directions of rotations of input and output (Both looking from respective output side) .

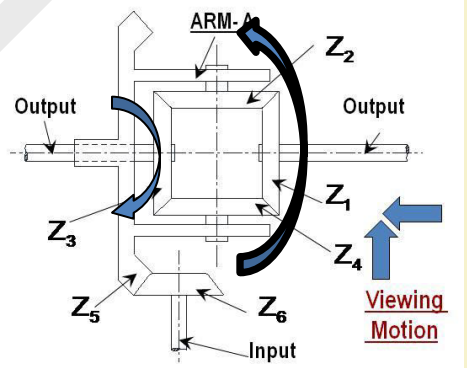
As magnitudes of ratios are more or less same (-49.25 and +50.2), the unit can be used as "Differential Unit".

Differential Gearing (Contd....) :

Tabular method to find differential output speed:

Operation	Speed (Direction of rotation – looking towards the arrow)						
	Planet Z_1	Planet Z_2	Planet Z_3	Planet Z_4	Gear 5	Arm Planet Carrier	Gear 6 (Input)
Arm fixed. Gear 1 given -1 Rotation.	-1	$-\frac{Z_1}{Z_2}$	$\frac{Z_1}{Z_2}\times\frac{Z_2}{Z_3}=\frac{Z_1}{Z_3}$	$+\frac{Z_1}{Z_4}$	0	0	0
The whole unit given +1 Rotation about Output-Output Axis (Except Z_6)	+1	+1	+1	+1	+1	+1	$-\frac{Z5}{Z_6}$
Total Resultant	0	$-\frac{Z_1}{Z_2}^{*}$	$1+\frac{Z_1}{Z_3}$	$+\frac{Z_1}{Z_4}^{*}$	+1	+1	$-\frac{Z5}{Z_6}$

Conventional differential unit:



In General :

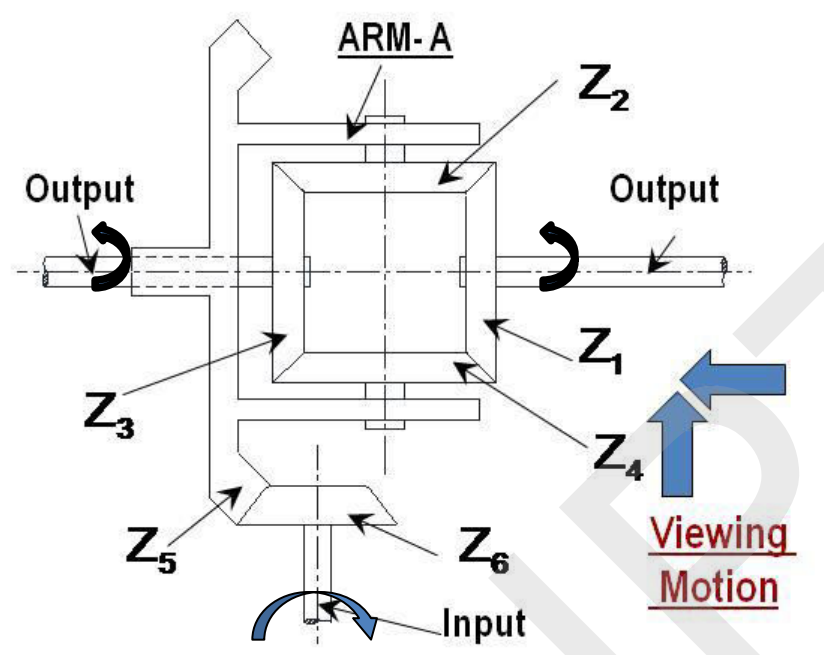
$$Z_1 = Z_2 = Z_3 = Z_4$$

However :

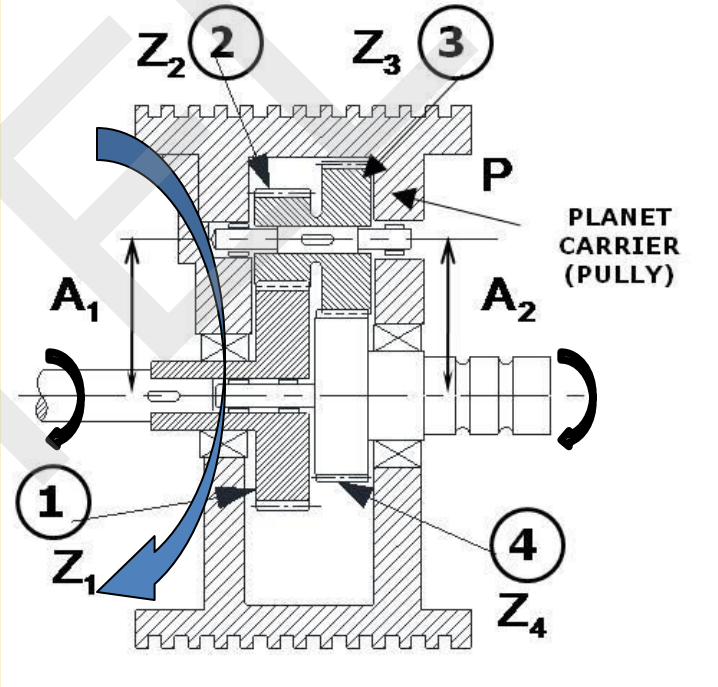
$Z_1 = Z_3$ **and** $Z_2 = Z_4$

***Looking from Input**

Differential Gearing:



Conventional differential unit:



Differential unit with Reverted Gearing:

Thank You

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Module-08

Internal, Epicyclic and Other Special Gearing

Lecture-42

Machine Tools Speed Change Gear Unit- I

NPTEL

Prof. Rathindranath Maiti
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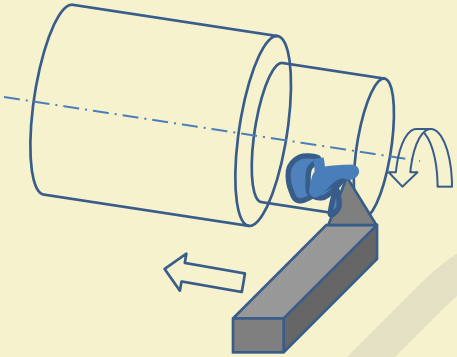
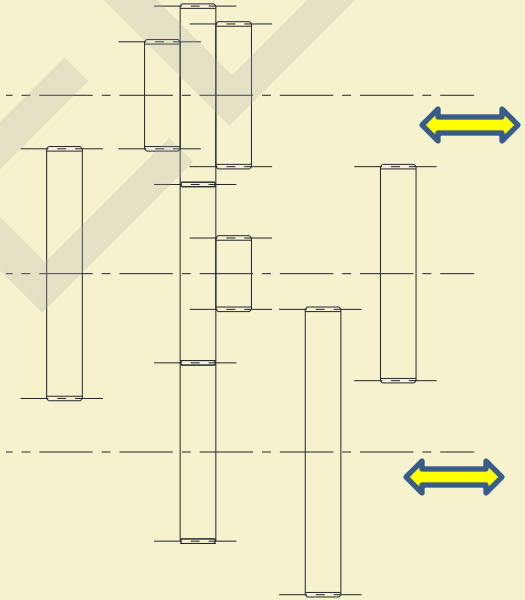
Outline of the Lecture

- Need for Speed Change Gear Box in Machine Tools
- Layout of Speed in Machine Tools
- Different Progression Series for Speeds
- Geometric Progression (GP)- General Rules, Stage Range
- Kinematic Advantage of GP Series
- Condition for Maximum Range in Transformation Stage

Need for Speed Change Gear Box in Machine Tools :



Turning in Lathe



Speed Change Gear Box in Lathe

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Layout of Speed in Machine Tools :

Layout of speeds are done commonly in one of the following series:

(i) Geometric Progression,

(ii) Arithmetic Progression,

(iii) Logarithmic Progression.

However, Geometric progression has several advantages over the other.

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Layout of Speed in Machine Tools (contd....) :

Layout of speeds in Geometric Progression:

The general rules:

$$n_f = n_1 \phi^{(f-1)}$$

Where,

n_f - **f th. Speed (rpm),**

n_1 - **Least Speed (rpm),**

ϕ - **Common ratio,**

f -**Number of steps.**

Hence,

$$\phi = \sqrt[(f-1)]{n_f / n_1} = \sqrt[(f-1)]{R_n}$$

Where,

R_n - **Stage range =** n_f / n_1

Layout of Speed in Machine Tools :

Layout of speeds in Geometric Progression:

Contd…..

Numerical Example:

Let, n_1 **- 10 rpm,**

n_f **- 450 rpm,**

and f **- 12 .**

Therefore,

ϕ **- Common ratio =**

$$\sqrt[(12-1)]{450/10} = \sqrt[11]{45} = 1.4$$

Main advantage of GP series is that common ratio is constant.

Much more varied speeds are available in comparison to AP Series.

More number of stocks can be machined keeping the cutting speed more or less constant.

Low cost.

Calculated speeds are: $10, 14, 20, 28, 40, 56, 80, 112, 160, 224, 315 \& 450.$

Kinematic Advantage of GP Series :

(i) Basic GP series- $A,\ A\phi,\ A\phi^2,\ .....A\phi^{(i-1)},\ A\phi^i,\ .....A\phi^{(2i-1)},\ A\phi^{2i},\ ....,\ A\phi^{(f-1)}$

(ii) With every i$^{th.}$ Term- $A,\ A\phi^i,\ A\phi^{2i},\ ....,\ A\phi^{mi}$

It is another GP series with common ratio- ϕ^i

(iii) Multiplying by a constant C with basic GP series-

$$C(A,\ A\phi,\ A\phi^2,\ .....A\phi^{(i-1)},\ A\phi^i,\ .....A\phi^{(2i-1)},\ A\phi^{2i},\ ....,\ A\phi^{(f-1)})$$

We get another GP series-

$$B,\ B\phi,\ B\phi^2,\ .....B\phi^{(i-1)},\ B\phi^i,\ .....B\phi^{(2i-1)},\ B\phi^{2i},\ ....,\ B\phi^{(f-1)}$$

Where, B=AC is the initial value but the common ratio is ϕ, same as in basic series.

Kinematic Advantage of GP Series (Contd…) :

(iv) Multiplying by a constant ϕ^m with basic GP series, the new series is -

$$A\phi^m,\ A\phi^{(m+1)},\ A\phi^{(m+2)},\ \ldots\ldots A\phi^{(m+i)},\ A\phi^{(m+i+1)},\ \ldots\ldots\ A\phi^{(m+2i-1)},$$

$$A\phi^{(m+2i)}\ \ldots\ldots,\ A\phi^{(m+f-1)}$$

where members of the series are shifted by m terms.

(v) Again in a GP series-

$$n_1,\ n_2,\ n_3,\ \ldots\ldots n_i,\ \ldots\ldots n_m,\ \ldots\ldots,\ n_f$$

$$n_i = n_1\phi^{(i-1)} = n_{e_1} l$$

$$and\ n_m = n_1\phi^{(m-1)} = n_{e_2} l$$

Then, $n_m / n_i = \phi^{(m-i)} = (n_{e_2} / n_{e_1})$

To develop a preferred GP series this rule is particularly useful.

Condition for Maximum Range in Transformation Stage :

Let in any transformation stage of p subdivisions,

$$n_j = n_{in} \frac{Z_j}{Z_j'}$$

Where, n_j **- Output Speed (rpm) of the stage,**

n_{in} **- Input Speed (rpm) of the stage,**

Z_j **- Gear tooth number/gear size/pulley size on input shaft,**

$[j = 1, 2, ..., p]$

Z_j' **- Mating Gear tooth number/gear size/pulley size on output shaft.**

$[j = 1, 2, ..., p]$

Condition for Maximum Range in Transformation Stage (contd….) :

The transformation ratios are:

$$\frac{n_j}{n_{in}} = \frac{Z_j}{Z_j'} = U_j \qquad [j = 1, 2, ..., p]$$

Let, $$\frac{Z_1}{Z_1'} < \frac{Z_2}{Z_2'} < \frac{Z_3}{Z_3'} < ..... < \frac{Z_p}{Z_p'}$$

Hence, $U_{j_{max}} = \frac{Z_p}{Z_p'}$ **and,** $U_{j_{min}} = \frac{Z_1}{Z_1'}$

Condition for Maximum Range in Transformation Stage (contd....) :

With respect to GP series transformation ratios are:

$$U_1 : U_2 : U_3 : \ldots\ldots\ldots : U_j = 1 : \phi : \phi^2 : \ldots\ldots : \phi^{(p-1)=x}$$

Hence, $$\frac{U_{j_{\max}}}{U_{j_{\min}}} = \phi^x = R_s$$

Where, R_s **= Stage Range.**

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Condition for Maximum Range in Transformation Stage (contd....) :

However, (a) due to limitation of gear pitch line velocity and

(b) also for space limitation,

The transformation ratio in a speed change gear unit is limited by:

$$\frac{1}{4} \leq U_j \leq \frac{2}{1}$$

Or in other words, for a given stage:

$$R_{s_{max}} = \left(\frac{U_{j_{max}}}{U_{j_{min}}} \right)_{max} = \phi_{max}^{x} \leq 8$$

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1. Principles of Machine Tools, by G. C. Sen & A. Bhattacharya, New Central Book agency pvt. Ltd., Kolkata. 2nd. Edn. 1975. ISBN 81-7381-155-5.

Thank You

Module-08

Internal, Epicyclic and Other Special Gearing

Lecture-43

Machine Tools Speed Change Gear Unit- II

NPTEL

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Outline of the Lecture

- General Rules for Kinematic Design of M/C Tools Speed Change Gear Box
- Six Speed 2x3 Ray Type Design
- Possible Gear Arrangement and Initiation of Ray Diagram
- Development of Gear Arrangement and Ray Diagram
- Gear Selection through a Practical Example
- Real Available Speeds and Ideal Speeds.

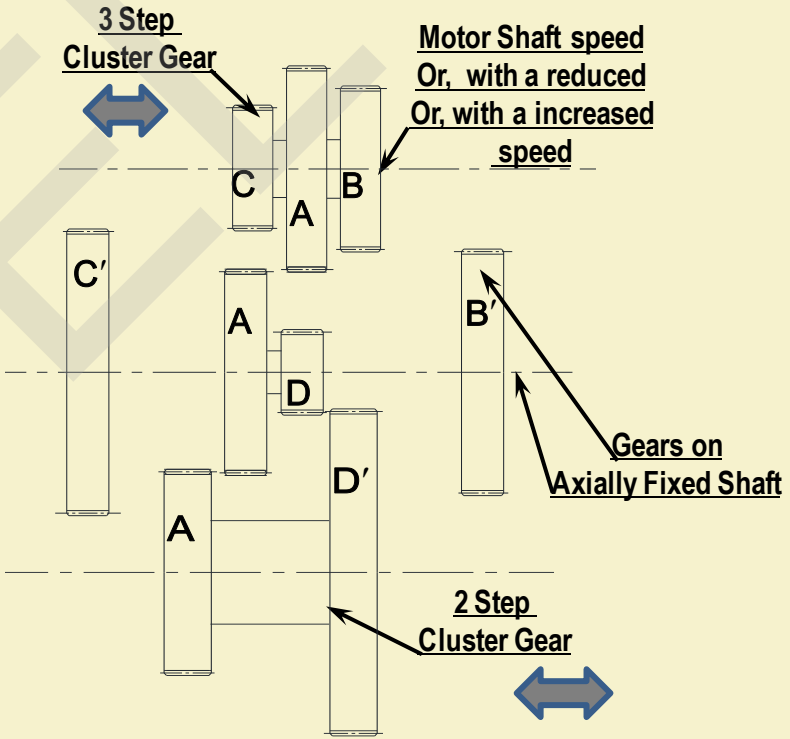
Gear Layout in M/C Tools Change Speed Gear Box :

General Rules

Rule 1 : One set of gears must be completely disengaged before the other set begins to come into mesh.

Rule 2 : The sum of the teeth of mating gears in a given stage must be same for same module in a clustered set.

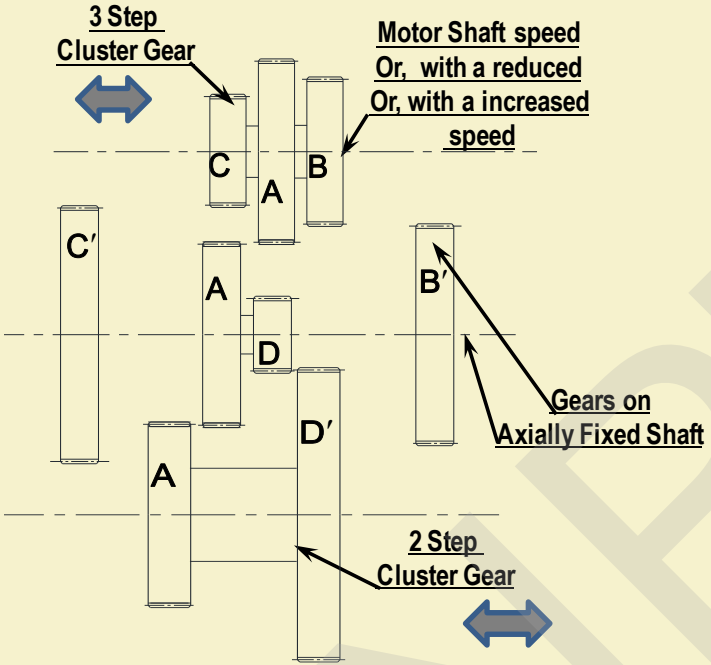
Rule 3 : The minimum difference between the numbers of teeth of adjacent gears must be four (4).



A Typical M/C Tools Change Speed Gear Box

Gear Layout in M/C Tools Change Speed Gear Box (contd…) :

General Rules (contd…)



A Typical M/C Tools Change Speed Gear Box

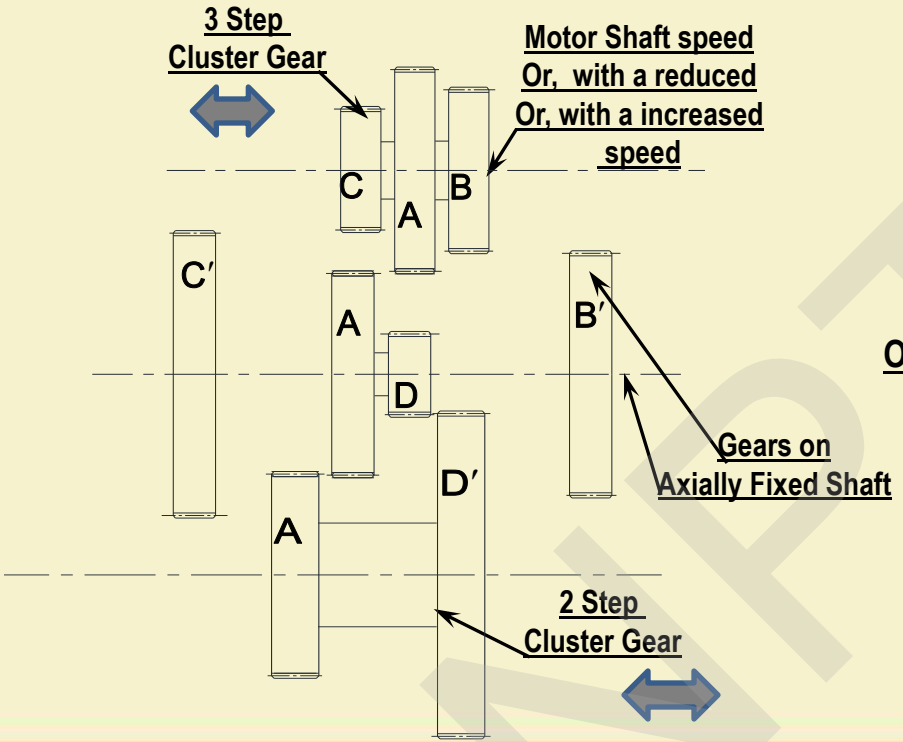
Rule 4 : The minimum number of teeth in a set of gears should be Greater than Equals to the critical number of teeth in order to avoid under cutting (interference). In case of 20° involute straight spur the critical number is 17.

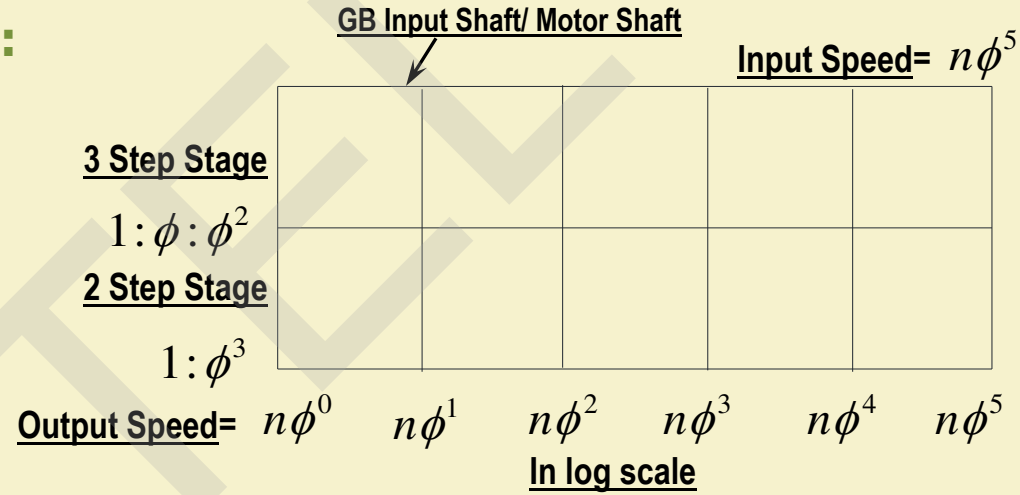
Rule 5 : The Maximum range for transformation is guided by:

$$\frac{1}{4} \le U_j \le \frac{2}{1}$$

Gear Layout in Six Speed Gear Box :

Arrangement A (2x3 Type) :



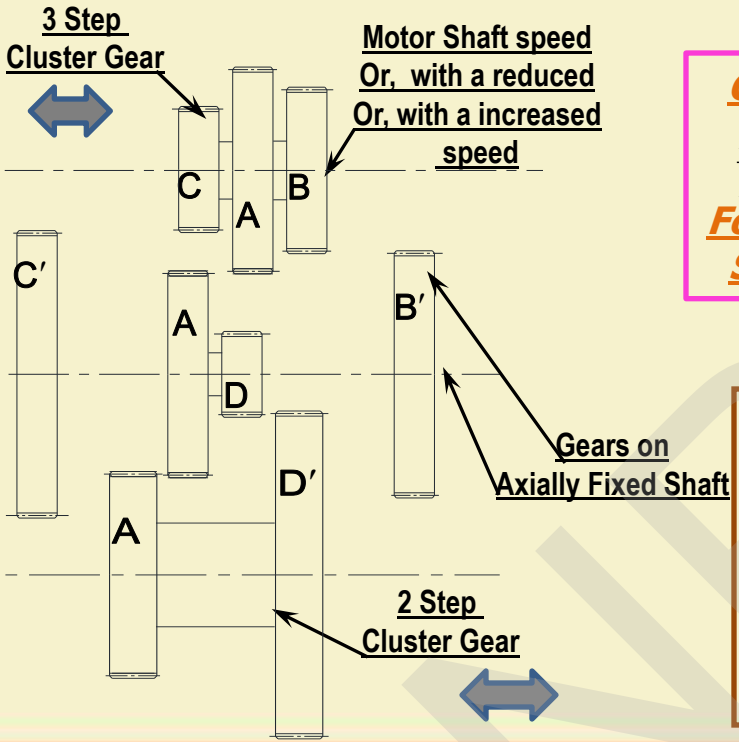


Ray Diagram for A Type 2 x 3 Ray Type
Six Speed Change M/C Tools Gear Box

NPTEL

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (contd.....) :



Clearly

$$\frac{Z_A}{Z_A} = 1$$

For Both Stages

3 Step Stage 1

2 Step Stage 1

Output Speed n_o =

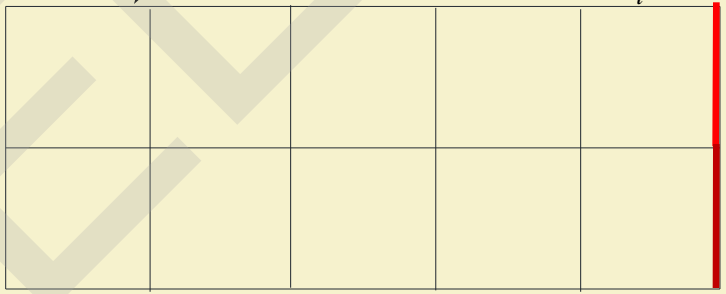
Total Transmission Ratio

$$i_{t1} = n_i / n_o = 1$$

$$= (1 \times 1)$$

GB Input Shaft/ Motor Shaft

Input Speed= $n_i = n\phi^5$



$n\phi^5$

In log scale

A
A
A
A

Gear Engagement

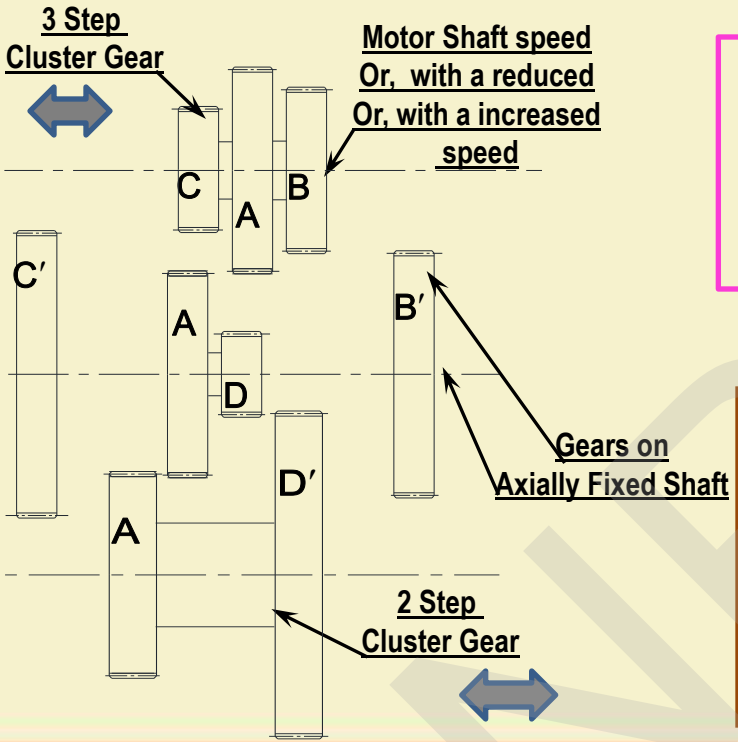
2 x 3 Ray Type

Ray Diagram for A Type

Six Speed Change M/C Tools Gear Box

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (contd.....) :



Clearly

$$\frac{Z_{B'}}{Z_B} = \phi \;\&\; \frac{Z_A}{Z_A} = 1$$

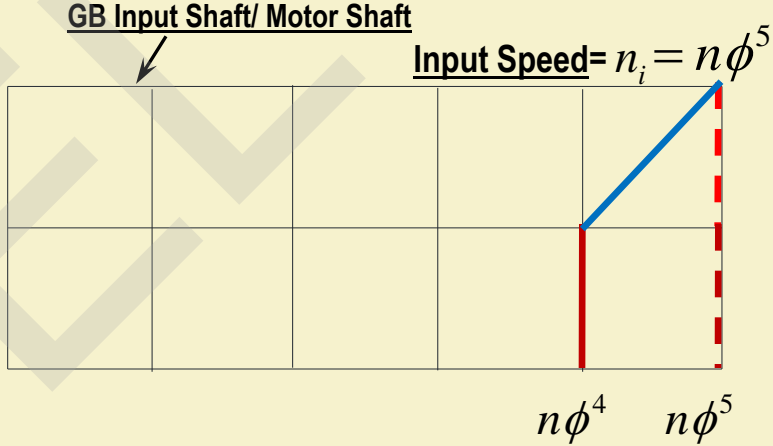
3 Step Stage ϕ

2 Step Stage 1

Output Speed n_o =

Total Transmission Ratio

$$i_{t1} = n_i / n_o = \phi = (1 \times \phi)$$

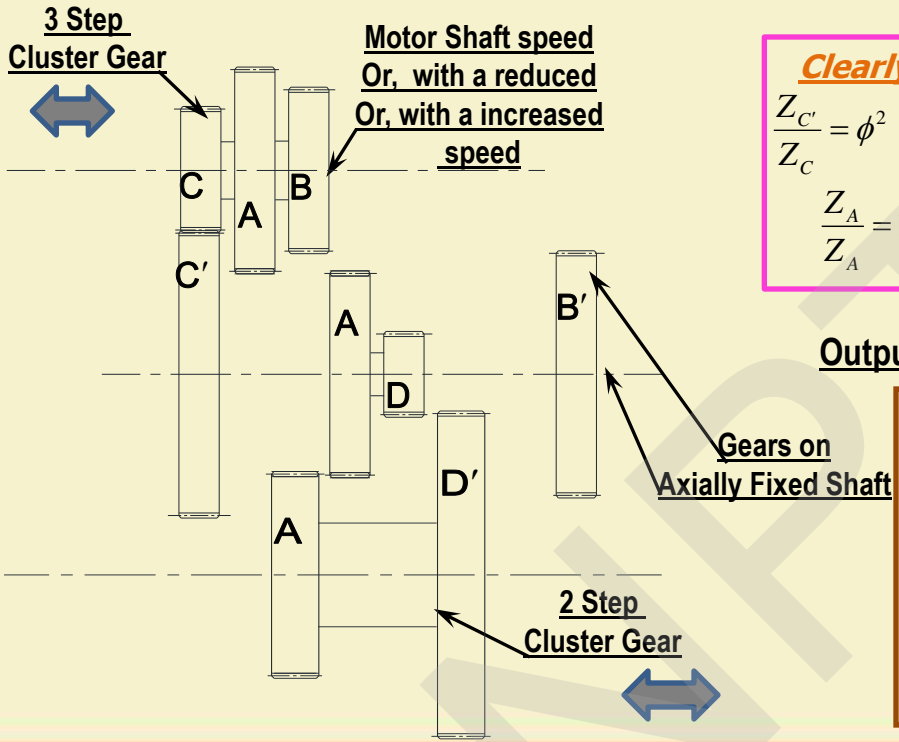


Ray Diagram for A Type — 2 x 3 Ray Type

Six Speed Change M/C Tools Gear Box

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (contd.....) :



Clearly

$$\frac{Z_{C'}}{Z_C} = \phi^2 \quad \& \quad \frac{Z_A}{Z_A} = 1$$

3 Step Stage ϕ^2

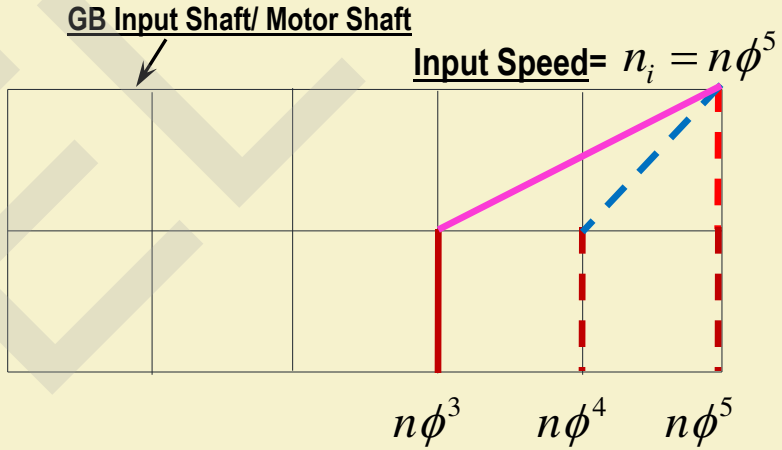
2 Step Stage 1

Output Speed n_o =

Total Transmission Ratio

$$i_{t1} = n_i / n_o = \phi^2$$

$$= (1 \times \phi^2)$$



In log scale

C
C′
A
A

Gear Engagement

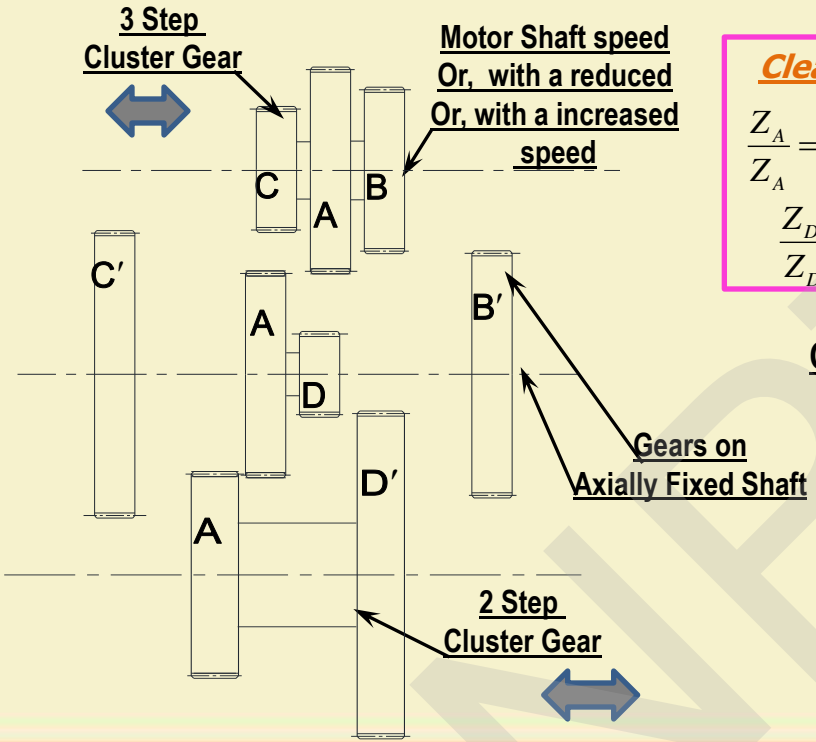
2 x 3 Ray Type

Ray Diagram for A Type
Six Speed Change M/C Tools Gear Box

NPTEL

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (contd.....) :



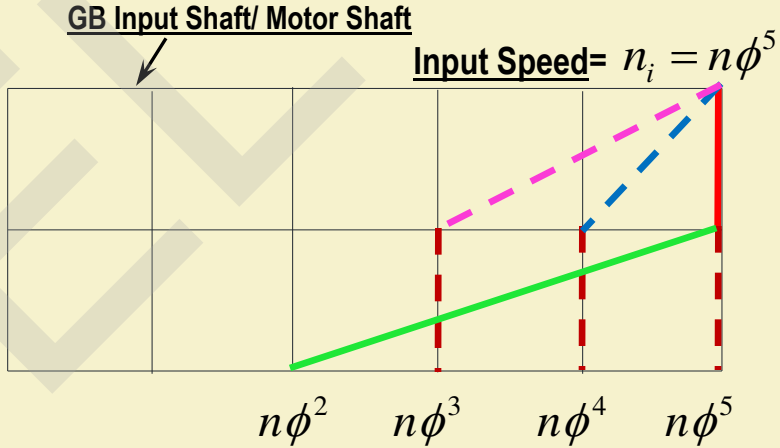
Clearly

$$\frac{Z_A}{Z_A} = 1 \quad \& \quad \frac{Z_{D'}}{Z_D} = \phi^3$$

3 Step Stage 1

2 Step Stage ϕ^3

Output Speed n_o =



In log scale

A
A
D
D′

Gear Engagement

2 x 3 Ray Type

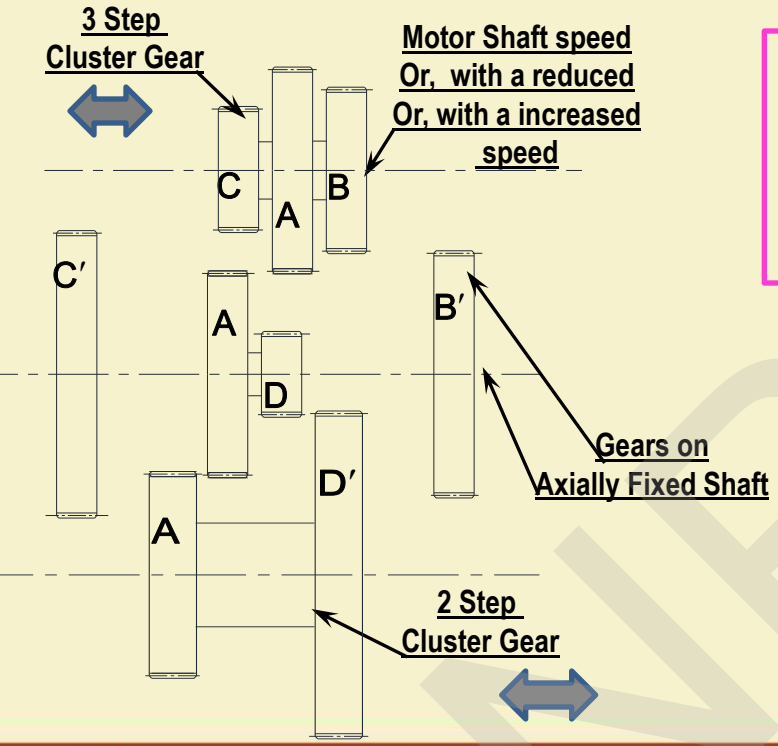
Total Transmission Ratio

$$i_{t1} = n_i / n_o = \phi^3$$
$$= (\phi^3 \times 1)$$

Ray Diagram for A Type
Six Speed Change M/C Tools Gear Box

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (contd.....) :



Clearly

$$\frac{Z_{B'}}{Z_B} = \phi \quad \& \quad \frac{Z_{D'}}{Z_D} = \phi^3$$

3 Step Stage ϕ

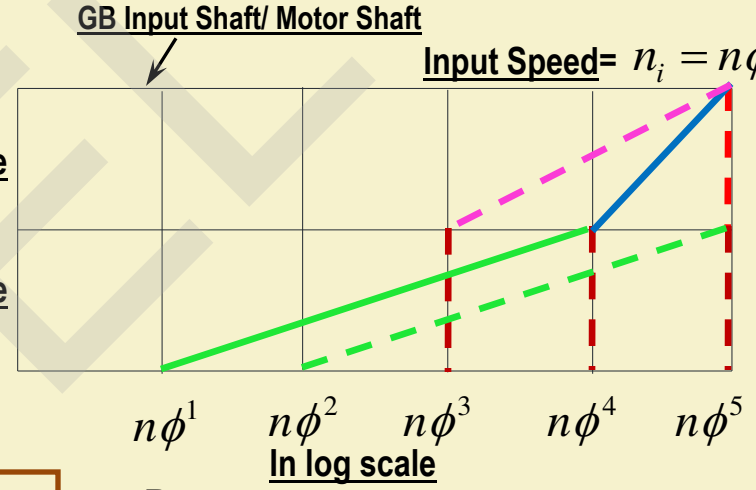
2 Step Stage ϕ^3

Output Speed n_o =

Total Transmission Ratio

$$i_{t1} = n_i / n_o = \phi^4$$

$$= (\phi^3 \times \phi)$$



B
B′
D
D′

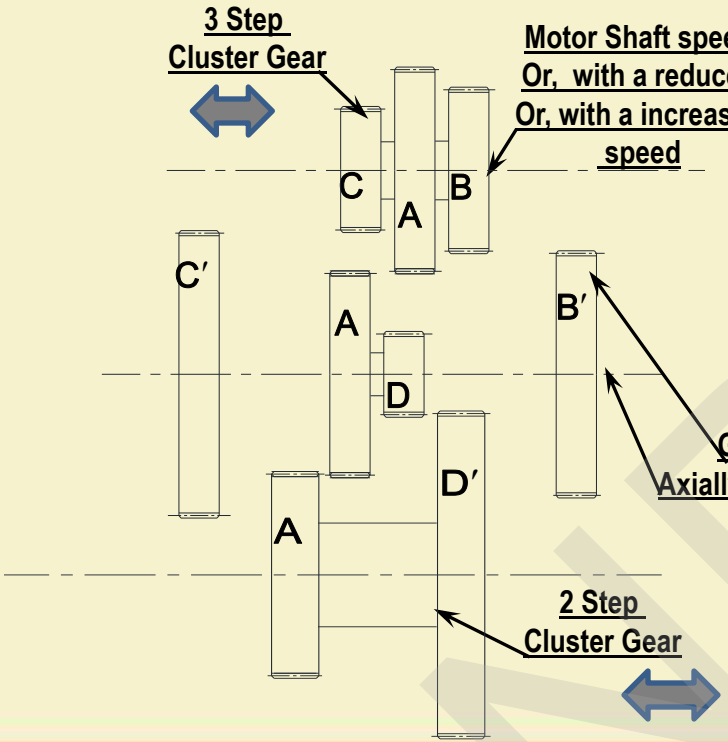
Gear Engagement

2 x 3 Ray Type

Ray Diagram for A Type
Six Speed Change M/C Tools Gear Box

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (contd.....) :

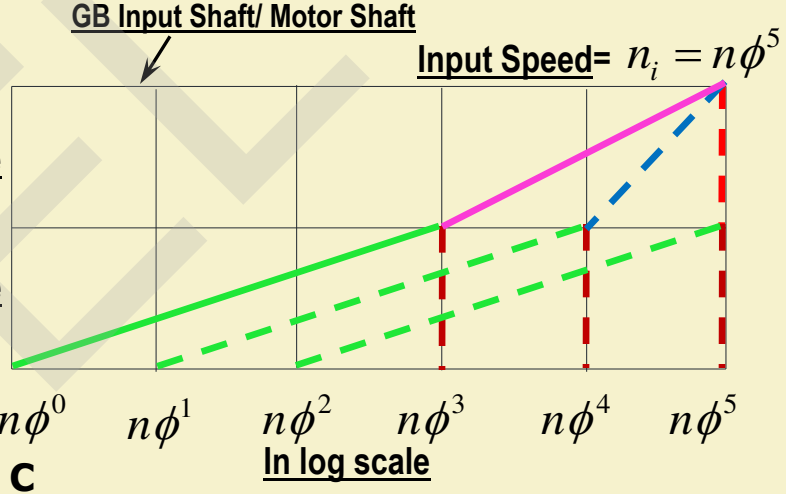


Clearly

$$\frac{Z_{C'}}{Z_C} = \phi^2 \quad \& \quad \frac{Z_{D'}}{Z_D} = \phi^3$$

3 Step Stage ϕ^2

2 Step Stage ϕ^3



Gear Engagement

2 x 3 Ray Type

Total Transmission Ratio

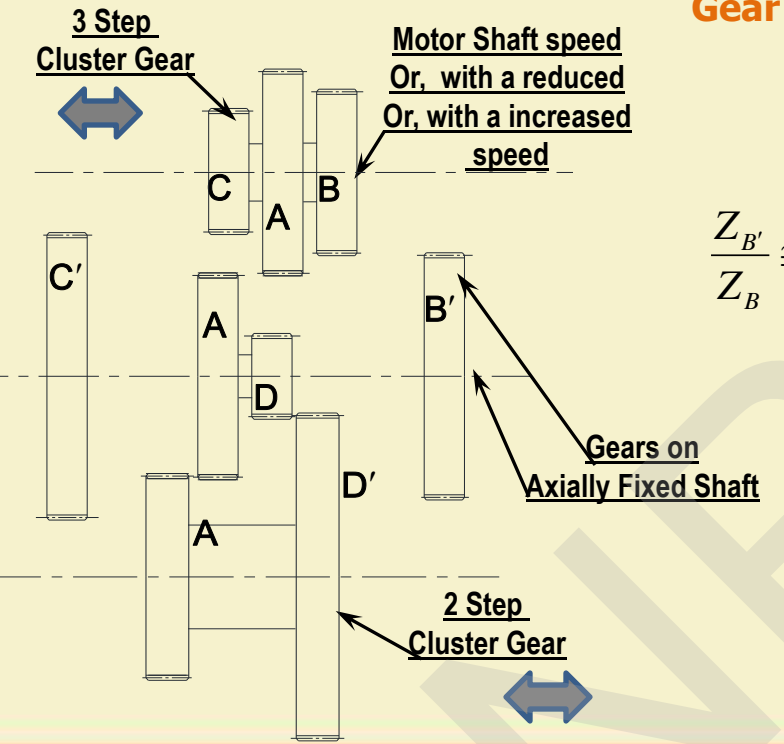
$$i_{t1} = n_i / n_o = \phi^5$$
$$= (\phi^3 \times \phi^2)$$

Ray Diagram for A Type
Six Speed Change
M/C Tools Gear Box

NPTEL

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (Final) :



Gear Ratios would be:

$$\frac{Z_A}{Z_A} = 1$$

$$\frac{Z_{B'}}{Z_B} \cong \phi$$

$$\frac{Z_{C'}}{Z_C} \cong \phi^2$$

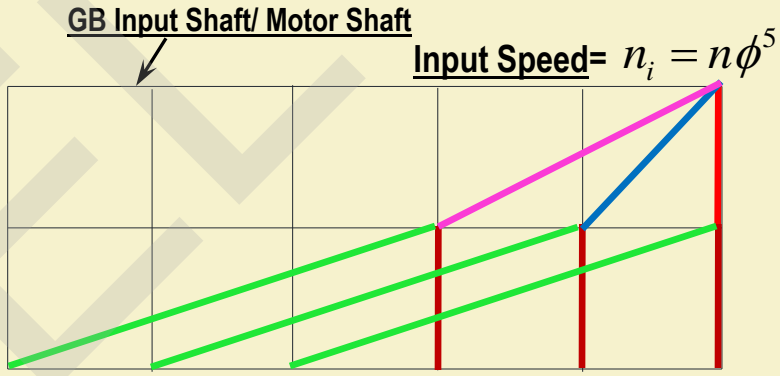
$$\frac{Z_{D'}}{Z_D} \cong \phi^3$$

3 Step Stage

$$1 : \phi : \phi^2$$

2 Step Stage

$$1 : \phi^3$$



Output Speed n_o = $n\phi^0$ $n\phi^1$ $n\phi^2$ $n\phi^3$ $n\phi^4$ $n\phi^5$

In log scale

C	B	A	C	B	A
C′	B′	A	C′	B′	A
D	D	D	A	A	A
D′	D′	D′	A	A	A

Gear Engagement

2 x 3 Ray Type

Ray Diagram for A Type

Six Speed Change M/C Tools Gear Box

NPTEL

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (contd.....) : Selection of Teeth Numbers :

Problem

Propose a 2x3 type (Arrangement A) six speeds machine tools change speed gear box with minimum output speed 100 rpm and maximum output speed 1450 rpm, which is also the output speed of the drive motor.

Show the ray diagram.

Propose the teeth number (20^{o} involute straight, uncorrected and of equal module).

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (contd.....) : Selection of Teeth Numbers (contd.....) :

Solution :

Given :

f -Number of steps= 6.

n_f -f th. Speed (rpm)
= Input Motor Speed, n_i = 1450 rpm,

n_1 - Least Speed (rpm) = 100 rpm,

To find Teeth numbers of Gears in 2x3 type (Arrangement A) six speeds machine tools change speed gear box.

Tooth would be 20° involute straight, uncorrected and of equal module.

ϕ - Common ratio is calculated as:

$$\phi = \sqrt[(f-1)]{n_f / n_1} = \sqrt[(f-1)]{R_n}$$

Speed Range : $$R_n = \frac{n_f}{n_1} = \frac{n_i}{n_{o(\min)}}$$

$$\therefore \quad R_n = \frac{1450}{100} = 14.50$$

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A :

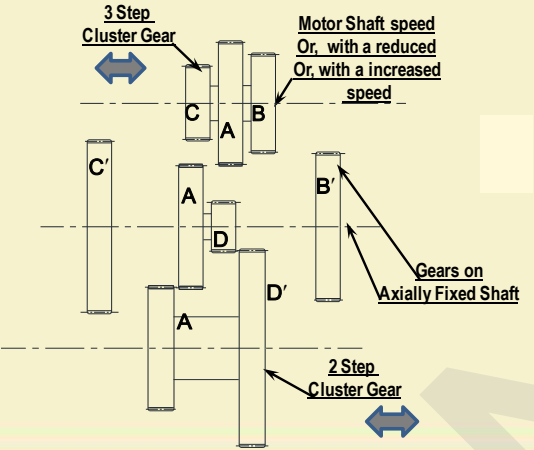
Solution (contd.....) :

Selection of Teeth Numbers (contd.....) :

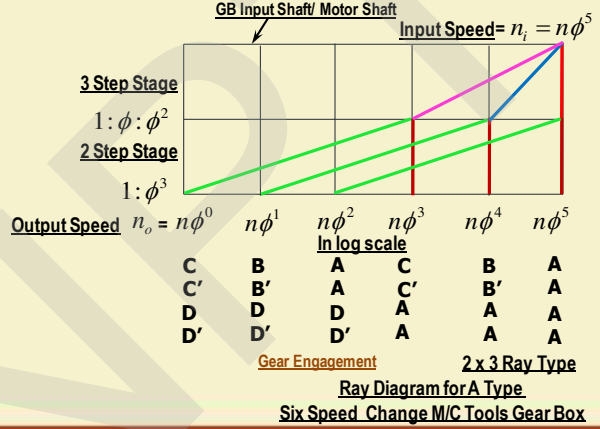
Common ratio : $\phi = \sqrt[(f-1)]{R_n} = \sqrt[(6-1)]{14.5} = 1.70715 = 1.707$

Therefore, ideal six speeds are : 100, 170.7, 291.4, 497.5, 849.4, 1450

The Gear Box will be as Follows:



The Ray Diagram and Gear Engagement Sequences:



We have to select Number of Teeth of Gears such that:

$$\frac{Z_A}{Z_A} = 1, \qquad \frac{Z_{B'}}{Z_B} \cong \phi = 1.707$$

$$\frac{Z_{C'}}{Z_C} \cong \phi^2 = 2.914 \qquad \frac{Z_{D'}}{Z_D} \cong \phi^3 = 4.975$$

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A : Selection of Teeth Numbers (contd.....) :

Solution (contd.....) :

Obviously Gear D will be of smallest size.

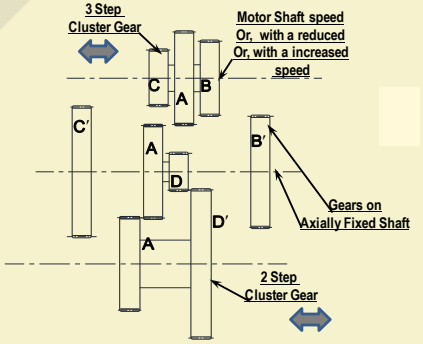
For 20° involute straight and uncorrected Precision (high speed) gear Recommended minimum teeth number is 18.

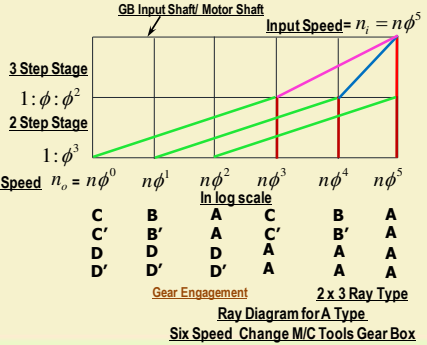
Let $Z_D = 20$ **As** $\frac{Z_{D'}}{Z_D} \cong \phi^3 = 4.975$ **then** $Z_{D'} \cong 20 \times 4.975 = 99.5$

Let $Z_{D'} = 100$

Now from Centre Distance (CD) Point of View (Following Rule)

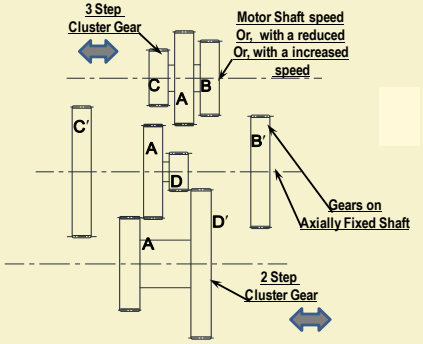
$2Z_A = Z_D + Z_{D'} = 20 + 100 = 120,$ **Therefore,** $Z_A = 120/2 = 60$

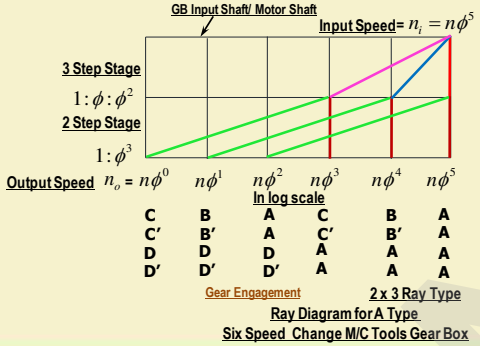




Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A :





Solution (contd.....) :

Selection of Teeth Numbers (contd.....) :

Now in 3 step stage also CD should be same (as gear A is there).

Therefore, $Z_B + Z_{B'} = Z_C + Z_{C'} = 2Z_A = 120$

Again $\frac{Z_{B'}}{Z_B} \cong \phi = 1.707$ **and** $\frac{Z_{C'}}{Z_C} \cong \phi^2 = 2.914$,

Solving for Teeth numbers,

$Z_B \cong 120/2.707 = 44.33$,

Let $Z_B = 44$,

Therefore, $Z_{B'} = 120 - 44$

$= 76$

And, $Z_c \cong 120/3.914 = 30.659$

Z_c **Could be take as 31, however it is taken as <u>30</u> and**

$Z_{c'} = 90$

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A :

Selection of Teeth Numbers (contd.....) :

Solution (contd.....) :

The Teeth numbers are finalized as:

$$Z_A = 60,\quad Z_B = 44,\quad Z_{B'} = 76,\quad Z_C = 30,\quad Z_{C'} = 90,\quad Z_D = 20 \;\&\; Z_D = 100$$

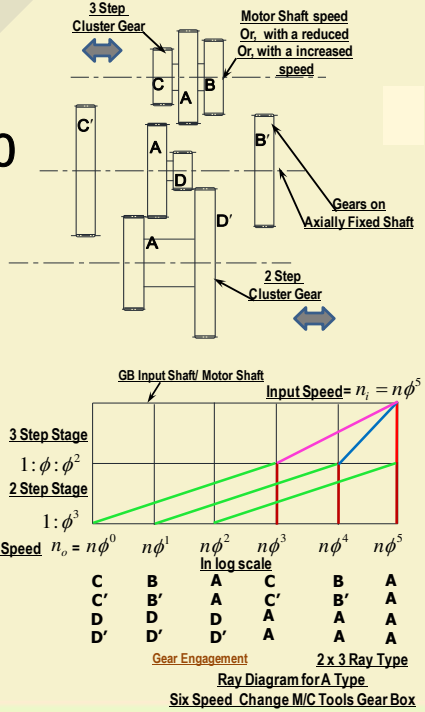
Ideal Gear Ratios are:

$$\frac{Z_A}{Z_A} = 1,\quad \frac{Z_{B'}}{Z_B} \cong \phi = 1.707 \qquad \frac{Z_{C'}}{Z_C} \cong \phi^2 = 2.914 \;\&\; \frac{Z_{D'}}{Z_D} \cong \phi^3 = 4.975$$

Real Gear Ratios are:

$$\frac{Z_A}{Z_A} = 1,\qquad \frac{Z_{B'}}{Z_B} = \frac{76}{44} = \phi' = 1.727$$

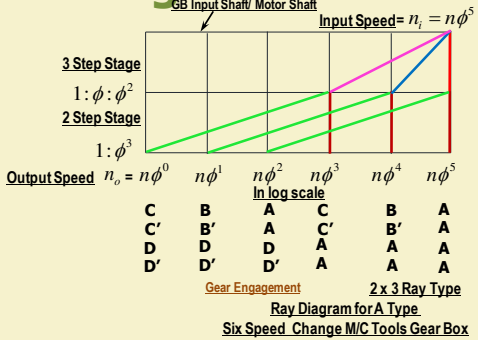
$$\frac{Z_{C'}}{Z_C} = \frac{90}{30} = (\phi')^2 = 3 \;\&\; \frac{Z_{D'}}{Z_D} = \frac{100}{20} = (\phi')^3 = 5$$



Gear Layout in Six Speed Gear Box (contd.....) :

Solution (contd.....) :

Arrangement A :



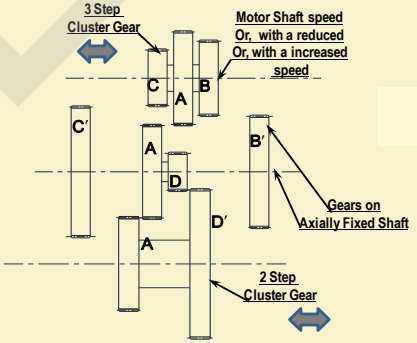
Selection of Teeth Numbers (contd.....) :

Real Gear Ratios are:

$$\frac{Z_A}{Z_A} = \frac{60}{60} = 1, \qquad \frac{Z_{B'}}{Z_B} = \frac{76}{44} = \phi' = 1.727$$

$$\frac{Z_{C'}}{Z_C} = \frac{90}{30} = (\phi')^2 = 3 \quad \&$$

$$\frac{Z_{D'}}{Z_D} = \frac{100}{20} = (\phi')^3 = 5$$



Real Speeds are:

$$\frac{1450}{3\times5} \cong 96.67, \quad \frac{1450}{1.727\times5} \cong 168, \quad \frac{1450}{1\times5} = 290, \quad \frac{1450}{3\times1} = 483.4, \quad \frac{1450}{1.727\times1} = 839.6 \;\&\; \frac{1450}{1\times1} = 1450$$

Ideal Speeds in GP Series are:

100, 170.7, 291.4, 497.5, 849.4, 1450

There are deviations but all rules are satisfied.

Real speeds are accepted.
Due to deviations at higher speeds, change in machining speeds will not be much.

Thank You

Prof. Rathindranath Maiti
Mechanical Engineering Department

Module-08

Internal, Epicyclic and Other Special Gearing

Lecture-44

Machine Tools Speed Change Gear Unit- III

NPTEL

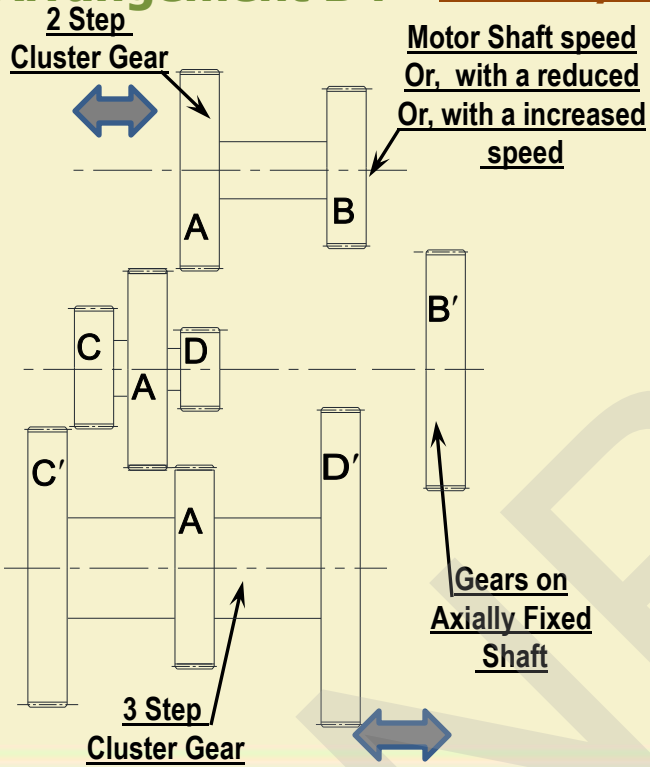
Prof. Rathindranath Maiti
Mechanical Engineering Department

Outline of the Lecture

- Six Speed 3x2 Ray Type Design
- Possible Gear Arrangement and Initiation of Ray Diagram
- Development of Gear Arrangement and Ray Diagram
- Gear Selection through a Practical Example
- Real Available Speeds and Ideal Speeds.
- Comparison between 2x3 Ray type with 3x2 Ray type Gearing.
- Concluding (Course Completion) Remarks.

Gear Layout in Six Speed Gear Box :

Arrangement B : 3 x 2 Ray Type



GB Input Shaft/ Motor Shaft

Input Speed, $n_i = n\phi^5$

2 Step Stage

$1 : \phi$

3 Step Stage

$1 : \phi^2 : \phi^4$

Output Speed, n_o	$n\phi^0$	$n\phi^1$	$n\phi^2$	$n\phi^3$	$n\phi^4$	$n\phi^5$

In log scale

B	A	B	A	B	A
B′	A	B′	A	B′	A
D	D	C	C	A	A
D′	D′	C′	C′	A	A

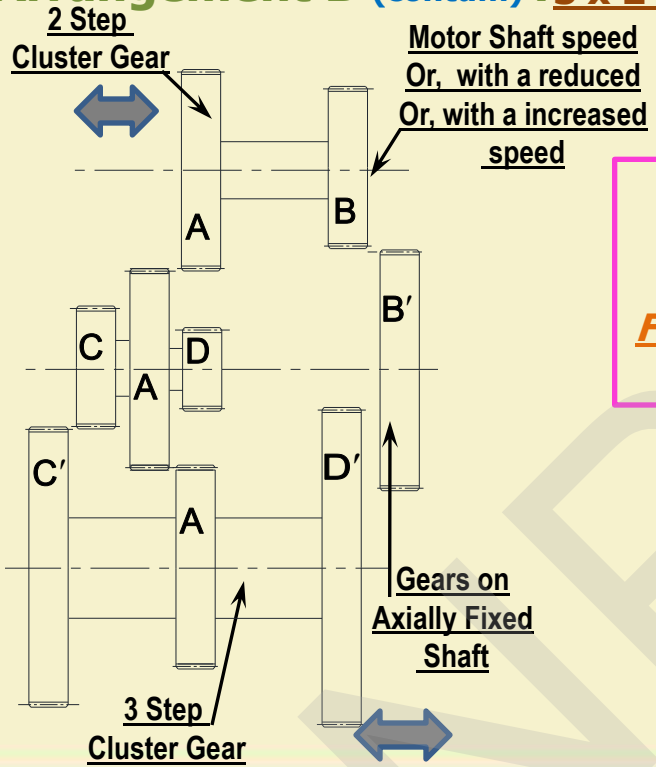
Gear Engagement

Ray Diagram for B (3 x 2 Ray) Type
Six Speed Change M/C Tools Gear Box

NPTEL

Gear Layout in Six Speed Gear Box (contd...) :

Arrangement B (contd...) : 3 x 2 Ray Type



Clearly

$$\frac{Z_A}{Z_A} = 1$$

For Both Stages

2 Step Stage

$$1 : \phi$$

3 Step Stage

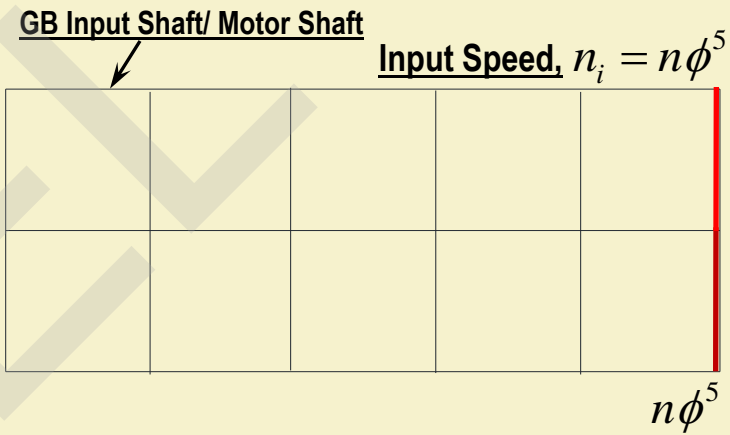
$$1 : \phi^2 : \phi^4$$

Output Speed, $n_o =$

Total Transmission Ratio

$$i_{t1} = n_i / n_o$$

$$= n\phi^5 / (1 \times 1)$$



In log scale

A
A
A
A

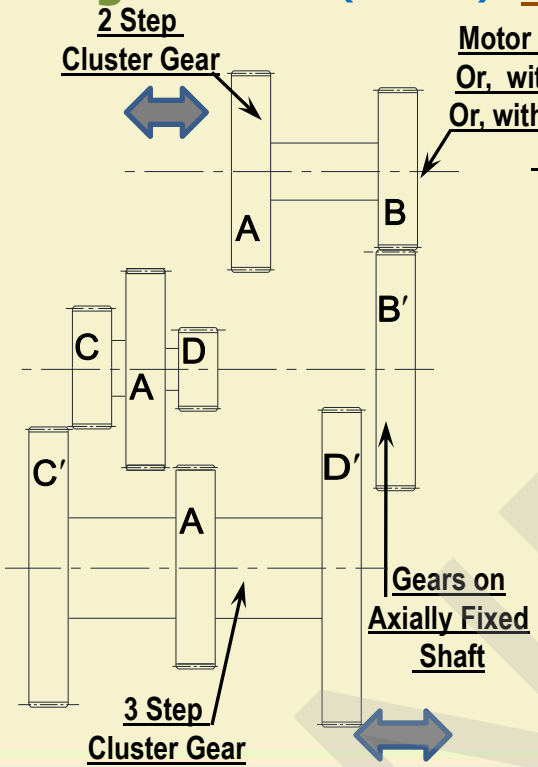
Gear Engagement

Ray Diagram for B (3 x 2 Ray) Type
Six Speed Change M/C Tools Gear Box

NPTEL

Gear Layout in Six Speed Gear Box (contd…) :

Arrangement B (contd…) : 3 x 2 Ray Type



Clearly

$$\frac{Z_{B'}}{Z_B} = \phi \quad \& \quad \frac{Z_A}{Z_A} = 1$$

2 Step Stage

$$1 : \phi$$

3 Step Stage

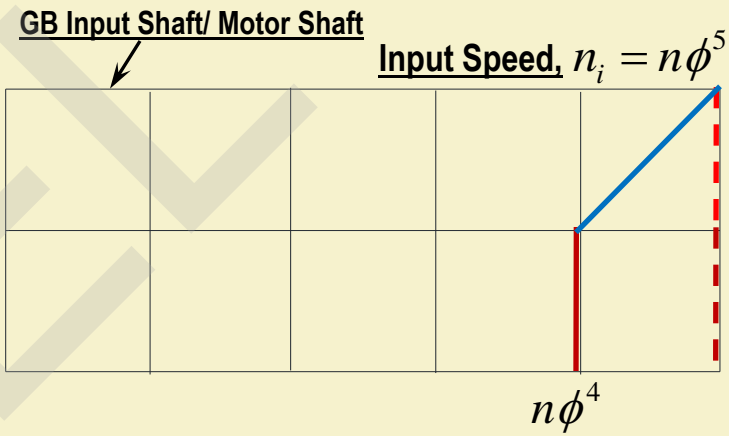
$$1 : \phi^2 : \phi^4$$

Output Speed, $n_o =$

Total Transmission Ratio

$$i_{t1} = \frac{n_i}{n_o} = \frac{n\phi^5}{n\phi^4}$$

$$\phi = (1 \times \phi)$$



In log scale

B
B′
A
A

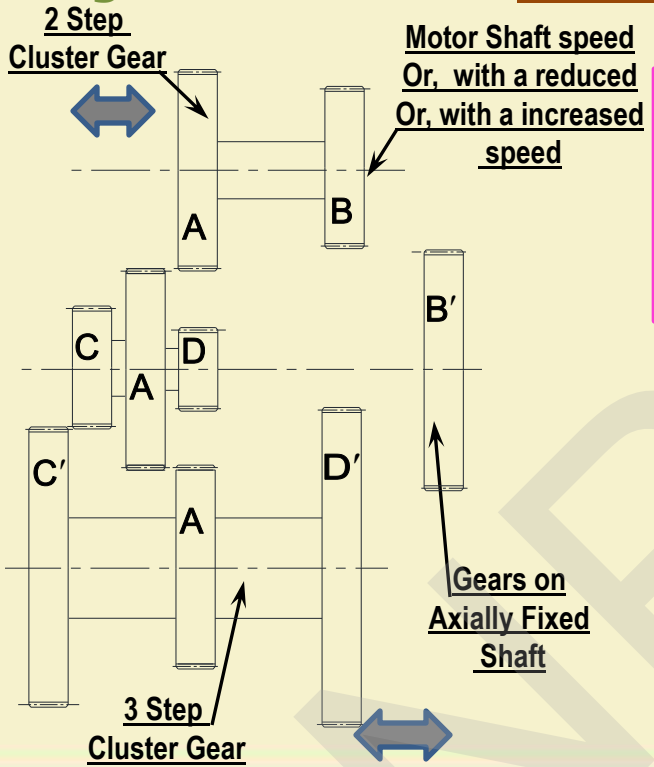
Gear Engagement

Ray Diagram for B (3 x 2 Ray) Type
Six Speed Change M/C Tools Gear Box

NPTEL

Gear Layout in Six Speed Gear Box (contd...) :

Arrangement B (contd...) : 3 x 2 Ray Type



Clearly

$$\frac{Z_A}{Z_A} = 1 \quad \&$$

$$\frac{Z_{C'}}{Z_C} = \phi^2$$

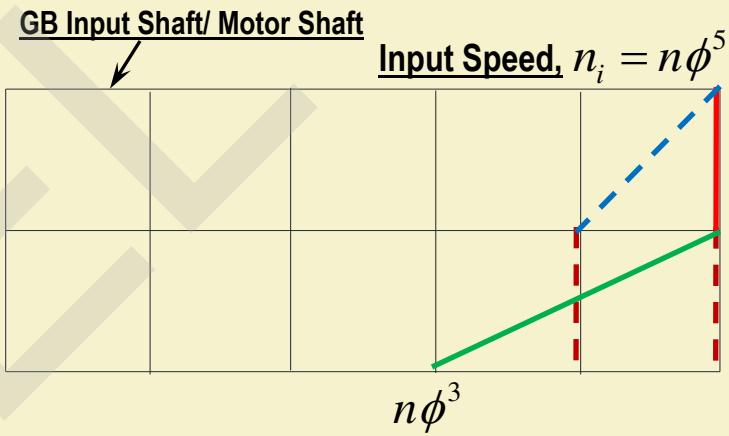
2 Step Stage

$$1 : \phi$$

3 Step Stage

$$1 : \phi^2 : \phi^4$$

Output Speed, $n_o =$



In log scale

A
A
C
C′

Gear Engagement

Total Transmission Ratio

$$i_{t1} = n_i / n_o = \phi^2$$

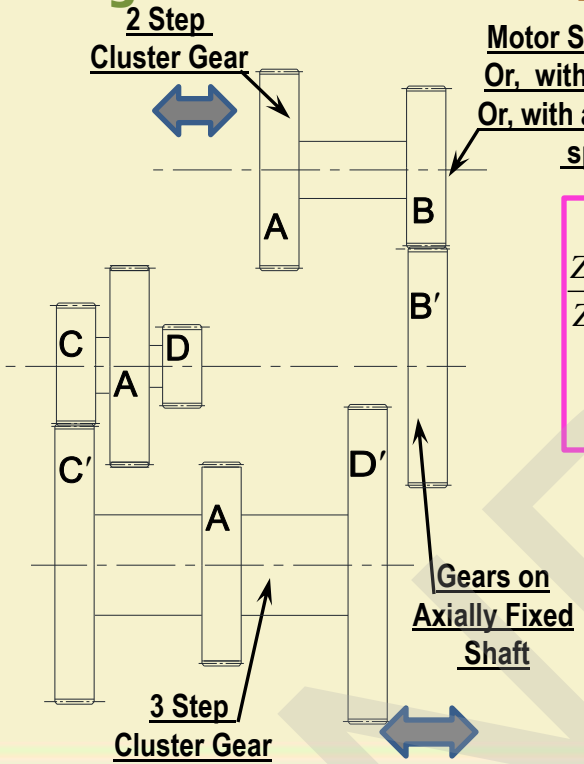
$$= (1 \times \phi^2)$$

Ray Diagram for B (3 x 2 Ray) Type Six Speed Change M/C Tools Gear Box

NPTEL

Gear Layout in Six Speed Gear Box :

Arrangement B : 3 x 2 Ray Type



Clearly

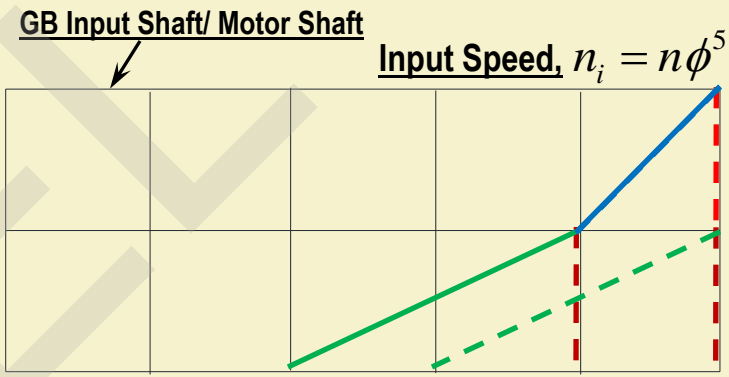
$$\frac{Z_{B'}}{Z_B} = \phi \quad \& \quad \frac{Z_{C'}}{Z_C} = \phi^2$$

2 Step Stage

$1 : \phi$

3 Step Stage

$1 : \phi^2 : \phi^4$



Output Speed, $n_o = n\phi^2$

In log scale

Total Transmission Ratio

$$i_{t1} = n_i / n_o = \phi^3$$

$$= (\phi \times \phi^2)$$

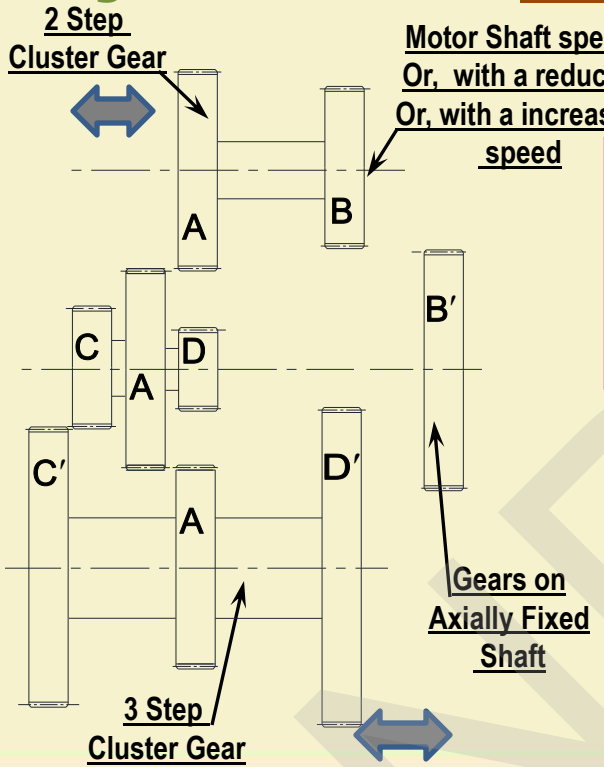
B
B′
C
C′

Gear Engagement

Ray Diagram for B (3 x 2 Ray) Type Six Speed Change M/C Tools Gear Box

Gear Layout in Six Speed Gear Box (contd…) :

Arrangement B (contd…) : 3 x 2 Ray Type



Clearly

$$\frac{Z_A}{Z_A} = 1 \quad \& \quad \frac{Z_{D'}}{Z_D} = \phi^4$$

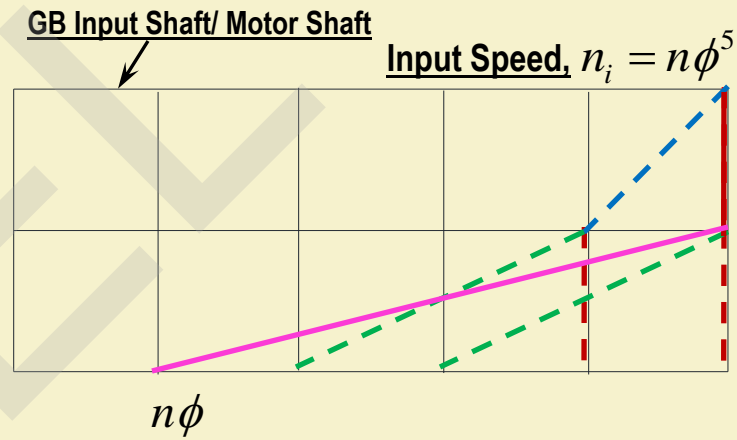
2 Step Stage

$$1 : \phi$$

3 Step Stage

$$1 : \phi^2 : \phi^4$$

Output Speed, $n_o =$



In log scale

Total Transmission Ratio

$$i_{t1} = n_i / n_o = \phi^4$$

$$= (\phi^4 \times 1)$$

A
A
D
D′

Gear Engagement

Ray Diagram for B (3 x 2 Ray) Type
Six Speed Change M/C Tools Gear Box

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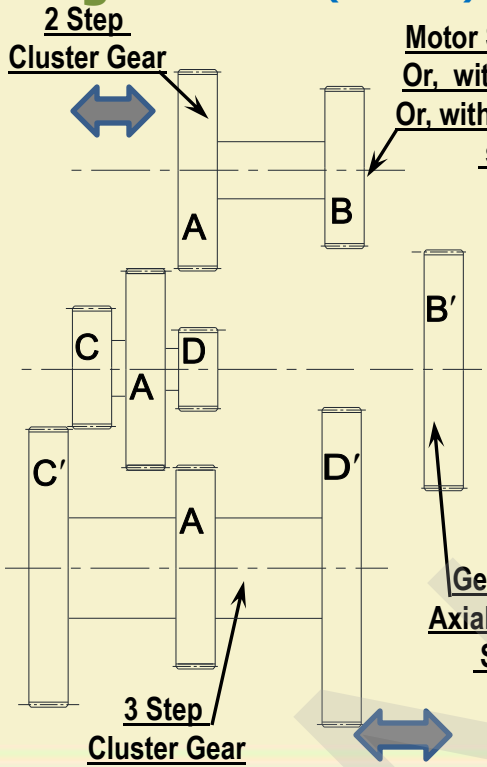
Gear Layout in Six Speed Gear Box (contd…) :

Arrangement B (contd…) : 3 x 2 Ray Type



Clearly

$$\frac{Z_{B'}}{Z_B} = \phi \quad \& \quad \frac{Z_{D'}}{Z_D} = \phi^4$$

Total Transmission Ratio

$$i_{t1} = n_i / n_o = \phi^5 = (\phi^4 \times \phi)$$

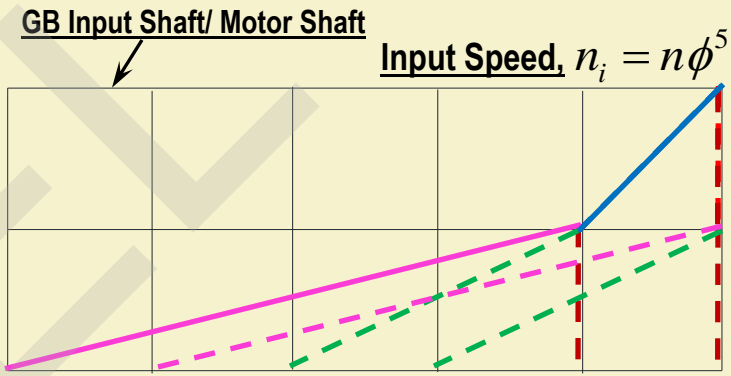
2 Step Stage

$$1 : \phi$$

3 Step Stage

$$1 : \phi^2 : \phi^4$$

Output Speed, $n_o = n\phi^0$



In log scale

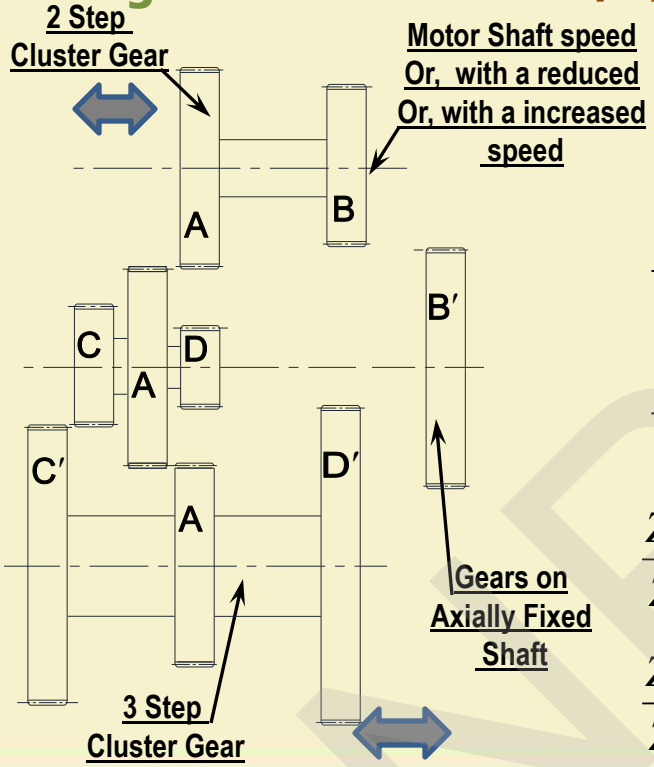
B
B′
D
D′

Gear Engagement

Ray Diagram for B (3 x 2 Ray) Type
Six Speed Change M/C Tools Gear Box

Gear Layout in Six Speed Gear Box :

Arrangement B : 3 x 2 Ray Type

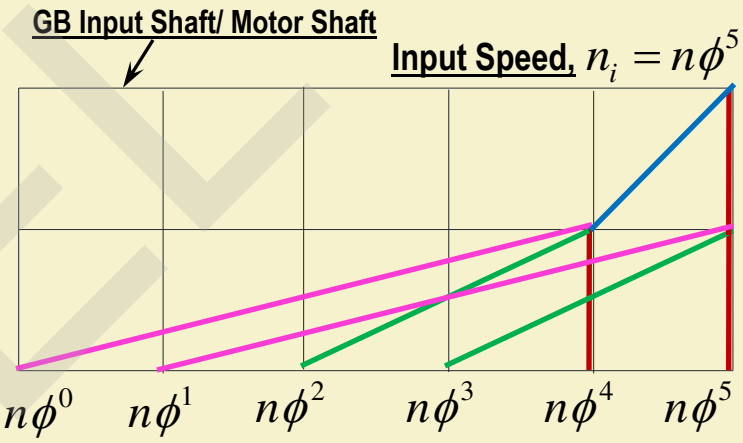


$$\frac{Z_A}{Z_A} = 1$$

$$\frac{Z_{B'}}{Z_B} = \phi$$

$$\frac{Z_{C'}}{Z_C} = \phi^2$$

$$\frac{Z_{D'}}{Z_D} = \phi^4$$



In log scale

B	A	B	A	B	A
B′	A	B′	A	B′	A
D	D	C	C	A	A
D′	D′	C′	C′	A	A

Gear Engagement

Ray Diagram for B (3 x 2 Ray) Type
Six Speed Change M/C Tools Gear Box

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Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement B (3x2 Ray Type) (contd.....) : Selection of Teeth Numbers :

Problem:

Propose a 3x2 ray type six speeds machine tools change speed gear boxes with minimum output speed 100 rpm and maximum output speed 1450 rpm, which is also the rpm of drive motor. Show the ray diagram. Propose the teeth number (20^{o} involute straight, uncorrected and of equal module). Compare 3x2 ray type gearing arrangement with 2x3 ray type arrangement.

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement B (3x2 Ray Type) (contd.....) :

Solution :

Selection of Teeth Numbers :

Given data:

f -Number of steps= 6.

n_f -f th. Speed (rpm)
= Input Motor Speed, n_i = 1450 rpm,

n_1 - Least Speed (rpm) = 100 rpm,

To find Teeth numbers of Gears in 3x2 type (Arrangement B) six speeds machine tools change speed gear box.

Tooth would be 20° involute straight, uncorrected and of equal module.

ϕ - Common ratio is calculated as:

$$\phi = \sqrt[(f-1)]{n_f / n_1} = \sqrt[(f-1)]{R_n}$$

Speed Range : $R_n = \frac{n_f}{n_1} = \frac{n_i}{n_{o(\min)}}$

$$\therefore \quad R_n = \frac{1450}{100} = 14.50$$

Gear Layout in Six Speed Gear Box (contd.....) :

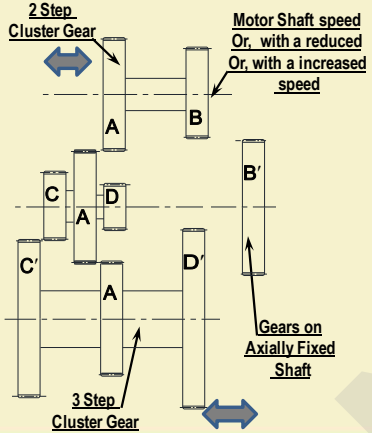
Arrangement B (3x2 Ray Type) (contd.....) :

Solution (contd.....) :

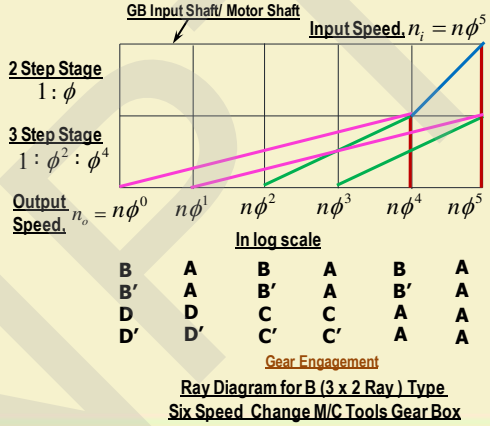
Common ratio : $\phi = \sqrt[(f-1)]{R_n} = \sqrt[(6-1)]{14.5} = 1.70715 = 1.707$

Therefore, ideal six speeds are : 100, 170.7, 291.4, 497.5, 849.4, 1450

The Gear Box will be as Follows:



The Ray Diagram and Gear Engagement Sequences:



Ray Diagram for B (3 x 2 Ray) Type Six Speed Change M/C Tools Gear Box

We have to select Number of Teeth of Gears such that:

$$\frac{Z_A}{Z_A} = 1, \qquad \frac{Z_{B'}}{Z_B} \cong \phi = 1.707$$

$$\frac{Z_{C'}}{Z_C} \cong \phi^2 = 2.914$$

$$\frac{Z_{D'}}{Z_D} \cong \phi^4 = 8.494$$

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement- B (3x2 Ray Type) (contd.....) :

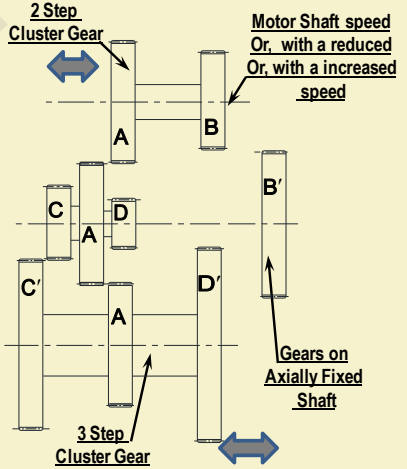
$$\frac{Z_{D'}}{Z_D} \cong \phi^4 = 8.494$$

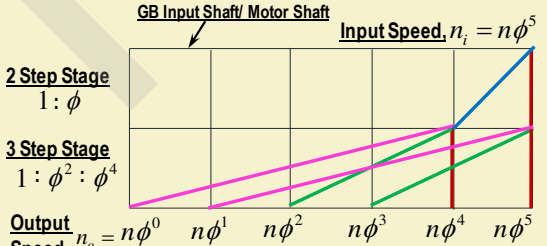
Is too high from both limitation in single reduction point of view as well as maximum Stage Range Point of view.

Arrangement- B, i.e., 3x2 Ray Type is to be used for lower common ratio ϕ, may be for 1.5 maximum.

However, for comparing Arrangements A & B current solution is continued.

Solution (contd.....) :





B	A	B	A	B	A
B′	A	B′	A	B′	A
D	D	C	C	A	A
D′	D′	C′	C′	A	A

Gear Engagement

Ray Diagram for B (3 x 2 Ray) Type Six Speed Change M/C Tools Gear Box

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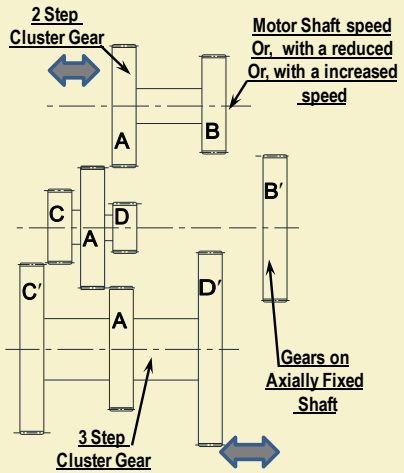
Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement- B (3x2 Ray Type) (contd.....) :



Solution (contd.....) :

Selection of Teeth Numbers :

Obviously Gear D will be of smallest size.

For 20° involute straight and uncorrected Precision (high speed) gear Recommended minimum teeth number is 18.

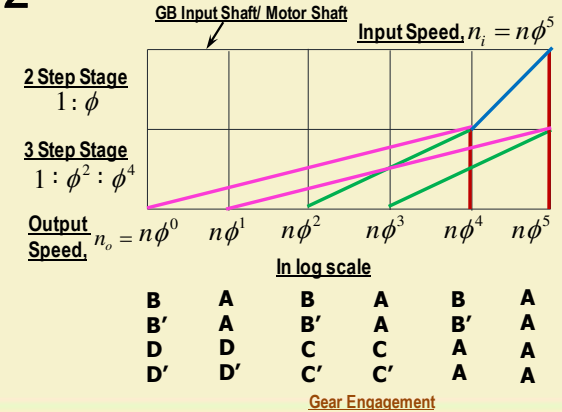
Let $Z_D = 18$ **As** $\frac{Z_{D'}}{Z_D} \cong \phi^4 = 8.494$ **then** $Z_{D'} \cong 18 \times 8.494 = 152.892$

Let $Z_{D'} = 152$

Now from Centre Distance (CD) Point of View (Following Rules), in this 3 step stage:

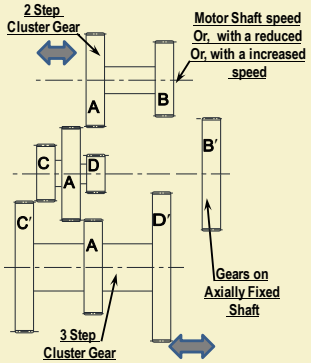
$$Z_D + Z_{D'} = Z_C + Z_{C'} = 2Z_A = 18 + 152 = 170,$$

Therefore, $Z_A = 170/2 = 85$



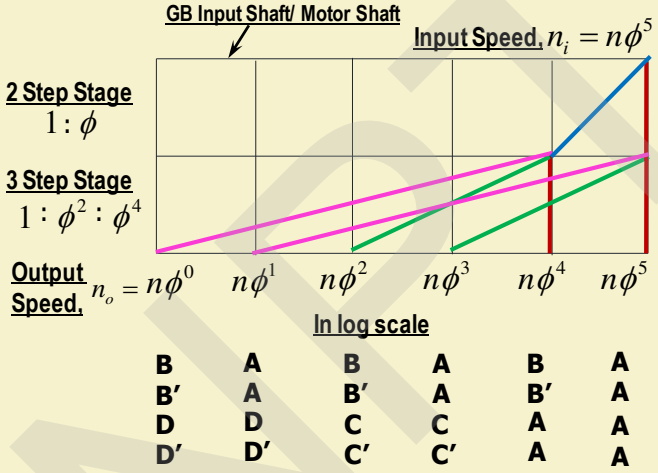
Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement- B (3x2 Ray Type) (contd.....) :



$$Z_A = 170/2 = 85$$

Again in this 3 step Stage $\frac{Z_{C'}}{Z_C} \cong \phi^2 = 2.914$, **and** $Z_C + Z_{C'} = 2Z_A = 170$



Solution (contd.....) :

Selection of Teeth Numbers :

Solving- $3.914Z_C = 170$,

i.e., $Z_C = 43.43$

Let $Z_C = 44$,

Therefore, $Z_{C'} = 170 - 44 = 126$,

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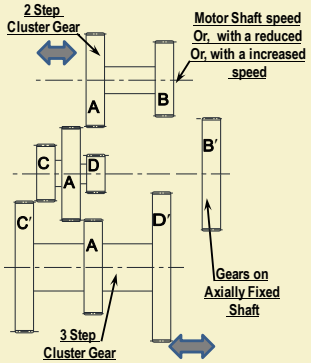
Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement- B (3x2 Ray Type) (contd.....) :

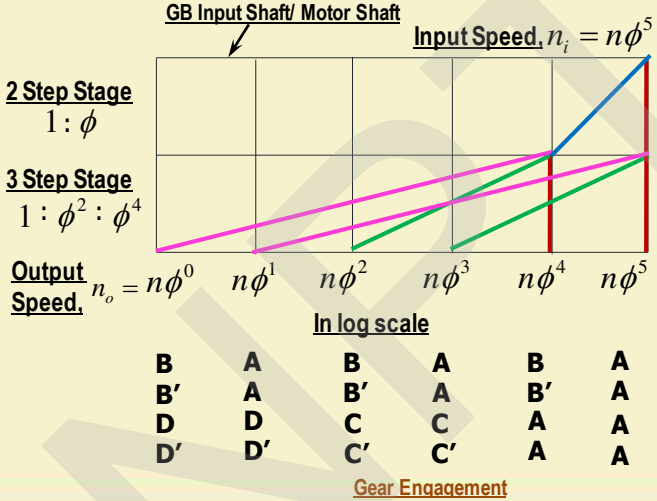


Solution (contd.....) :

Selection of Teeth Numbers :

Now in 2 step stage also CD should be same (as gear A is there).

Therefore, $Z_B + Z_{B'} = 2Z_A = 170,$



Again $\frac{Z_{B'}}{Z_B} \cong \phi = 1.707$

Solving, $Z_B = \frac{170}{2.707} = 62.8,$

Let $Z_B = 63,$

Therefore, $Z_{B'} = 170\text{-}63 = 107$

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Gear Layout in Six Speed Gear Box (contd.....) :

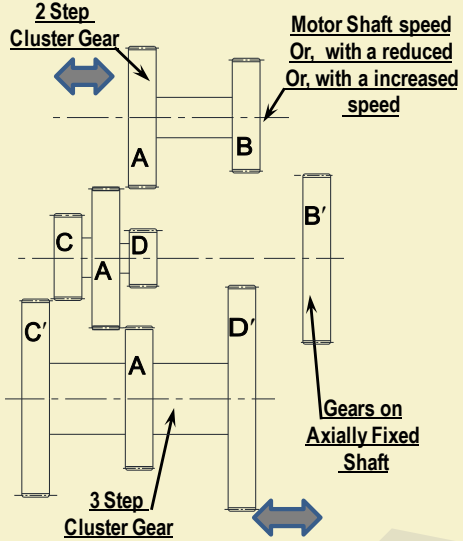
Arrangement- B (3x2 Ray Type) (contd.....) :

Solution (contd.....) :

Selection of Teeth Numbers :

Selected Teeth Numbers are :

$$Z_A = 85, \quad Z_B = 63, \quad Z_{B'} = 107, \quad Z_C = 44, \; Z_{C'} = 126, \; Z_D = 18 \;\; \& \;\; Z_{D'} = 152$$

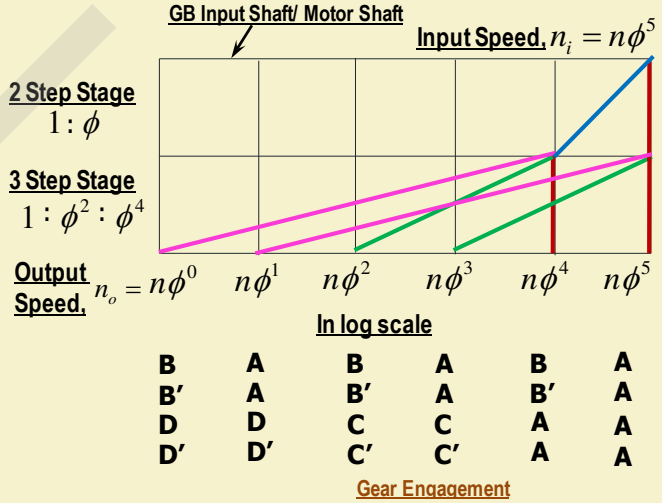


Gear Ratios are : (Real) **In comparison to: (Ideal)**

$$Z_{B'} / Z_B = 107/63 = 1.698, \quad \phi = 1.707$$

$$Z_{C'} / Z_C = 126/44 = 2.864, \quad \phi^2 = 2.914$$

$$Z_{D'} / Z_D = 152/18 = 8.44, \quad \phi^4 = 8.494$$



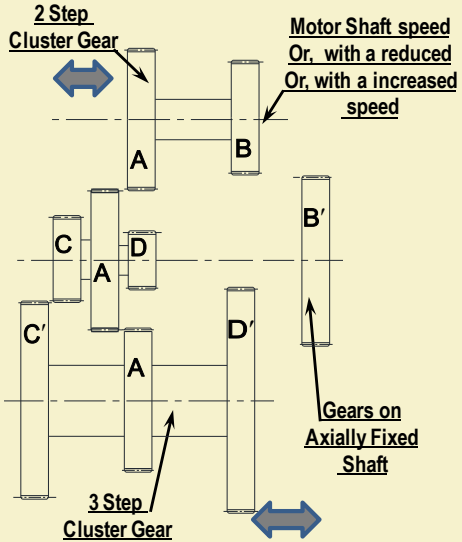
Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement- B (3x2 Ray Type) (contd.....) :

Solution (contd.....) :

Selection of Teeth Numbers :

Selected Teeth Numbers are : $Z_A = 85, Z_B = 63, Z_{B'} = 107, Z_C = 44, Z_{C'} = 126, Z_D = 18$ & $Z_{D'} = 152$



Ideal

2 Step Stage

$1 : \phi$

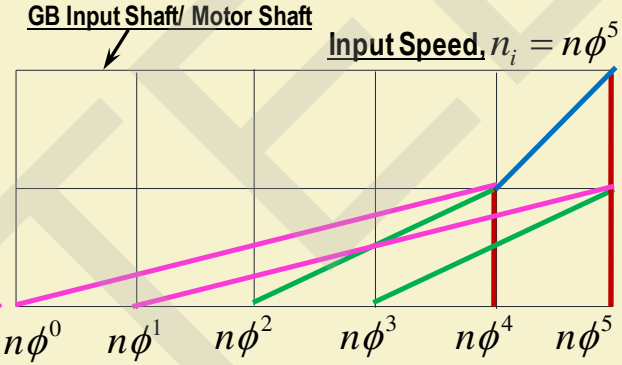
= 1 : 1.707

3 Step Stage

$1 : \phi^2 : \phi^4$

= 1 : 2.914:8.494

Output Speed, $n_o = n\phi^0 \quad n\phi^1 \quad n\phi^2 \quad n\phi^3 \quad n\phi^4 \quad n\phi^5$



Real

= 1 : 1.6984

= 1 : 2.864:8.44

In log scale

B	A	B	A	B	A
B′	A	B′	A	B′	A
D	D	C	C	A	A
D′	D′	C′	C′	A	A

Gear Engagement

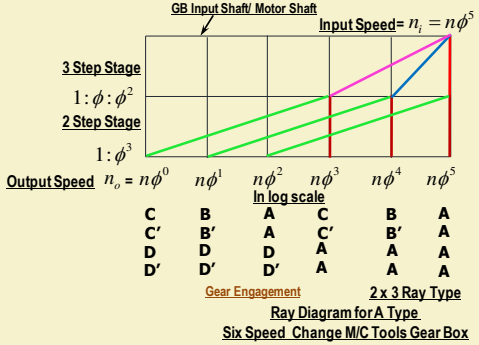
Real Speeds are : 101.2 171.8 298 506.3 853.75 1450

In comparison to Ideal Speeds in GP Series: 100 170.7 291.4 497.4 849.44 1450

Real available Speeds are Acceptable

Gear Layout in Six Speed Gear Box (contd.....) :

Arrangement A (recapitulation) :



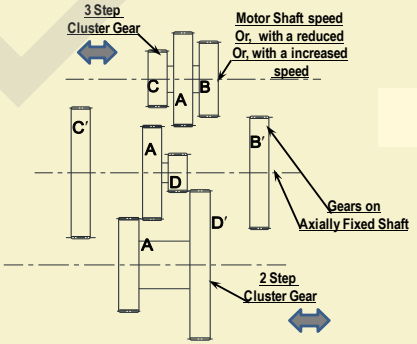
Comparison of Arrangement A with Arrangement B

Real Gear Ratios are:

$$\frac{Z_A}{Z_A} = 1, \qquad \frac{Z_{B'}}{Z_B} = \frac{76}{44} = \phi' = 1.727$$

$$\frac{Z_{C'}}{Z_C} = \frac{90}{30} = (\phi')^2 = 3 \quad \&$$

$$\frac{Z_{D'}}{Z_D} = \frac{100}{20} = (\phi')^3 = 5$$



Real Speeds are:

$$\frac{1450}{3\times5} \cong 96.67, \quad \frac{1450}{1.727\times5} \cong 168, \quad \frac{1450}{1\times5} = 290, \quad \frac{1450}{3\times1} = 483.4, \quad \frac{1450}{1.727\times1} = 839.6 \;\&\; \frac{1450}{1\times1} = 1450$$

Ideal Speeds in GP Series are:

100, 170.7, 291.4, 497.5, 849.4, 1450

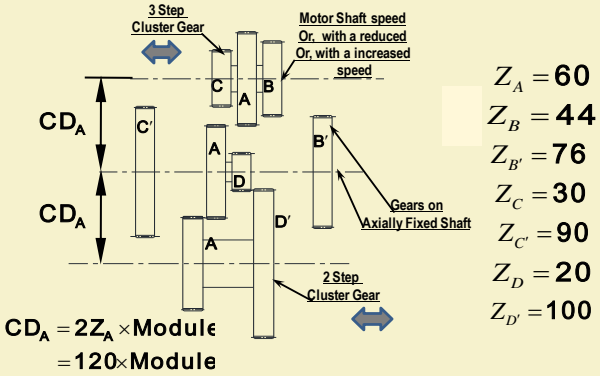
There are deviations but all rules are satisfied.

Real speeds are accepted.
Due to deviations at higher speeds, change in machining speeds will not be much.

Gear Layout in Six Speed Gear Box (contd.....) :

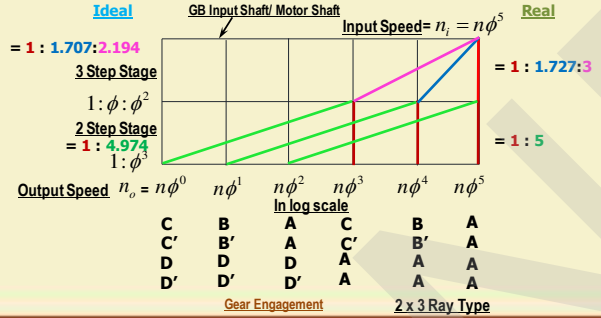
Comparison between Arrangement- A (2x3 Ray Type) & Arrangement- B (3x2 Ray Type) :

Arrangement- A (2x3 Ray Type)



$CD_A = 2Z_A \times \text{Module}$
$= 120 \times \text{Module}$

$Z_A = 60$
$Z_B = 44$
$Z_{B'} = 76$
$Z_C = 30$
$Z_{C'} = 90$
$Z_D = 20$
$Z_{D'} = 100$



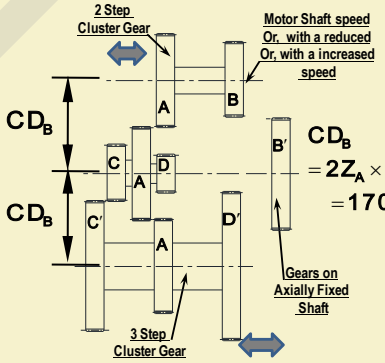
Speed (rpm)				
A (2x3 Ray Type)		Ideal (GP Series)	B (3x2 Ray Type)	
Deviation %	Real		Real	Deviation %
-3.33	96.67	100	101.2	+1.2
-1.58	168	170.7	171.8	+0.64
-0.05	290	291.4	298	+2.26
-2.8	483.4	497.4	506.3	+1.8
-1.15	839.6	849.4	853.7	+0.05
0.0	1450	1450	1450	0.0

Note: $CD_B > CD_A$

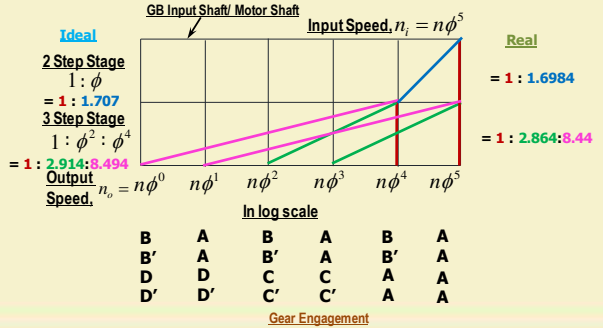
Even module is taken
2.5 mm for arrangement A
and 2 mm for arrangement B.

Arrangement- B (3x2 Ray Type)

$Z_A = 85$
$Z_B = 63$
$Z_{B'} = 107$
$Z_C = 44$
$Z_{C'} = 126$
$Z_D = 18$
$Z_{D'} = 152$



$CD_B = 2Z_A \times \text{Module}$
$= 170 \times \text{Module}$



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End of the lecture series and course

Thank You

Prof. Rathindranath Maiti
Mechanical Engineering Department